VOL. 23 No. 276

WIRELESS ENGINEER

SEPTEMBER 1946

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EDITORIAL

Simple Transmission Formula

UNDER this title H. T. Friis of the Bell Telephone Laboratories, published a note in the *Proc. I.R.E.* of May 1946. He makes some novel and interesting suggestions concerning the way in which the relation between the radiated and received power should be stated. The basic idea is that an aerial has an effective area which is defined, whether the aerial is actually used for transmitting or receiving, by assuming it to be used as a receiver of a linearly polarized plane wave. If P_0 is the power flow per unit area in the neighbourhood of the aerial, and P_r is the power available at the output terminals of the aerial then the effective area A_r is defined as P_r/P_0. An aerial can have different effective areas in different directions, and the effective area will be modified by variations of the load resistance into which the aerial feeds. In any transmission formula the areas must, of course, be those in the direction of transmission. If P_t be the power fed into the transmitting aerial at its input terminals and P_r the power available at the output terminals of the receiving aerial then

$$P_r/P_t = A_r A_t/d^2\lambda^2$$

where A_r and A_t are the effective areas of the two aerials and d their distance apart. Since the right-hand side of the formula has dimensions L^4/L^4, it is immaterial what units are employed, so long as they are the same throughout.

Considering a small dipole of negligible resistance and length l with a load resistance equal to its radiation resistance, i.e. a small uniform current element, situated in an electric field of strength $\mathscr{E}$ r.m.s. volts per unit length, then

$$I = \frac{\mathscr{E}l}{R_l + R_{rad}} = \frac{\mathscr{E}l}{2R_{rad}} \text{ and}$$

$$P_r = I^2R_l = \frac{\mathscr{E}^2l^2}{4R_{rad}} \text{ watts.}$$

Now for such an element

$$R_{rad} = 80\pi^2l^2/\lambda^2 \text{ ohms}$$

hence $$P_r = \frac{\mathscr{E}^2\lambda^2}{320\pi^2} \text{ watts}$$

and since $P_0 = \mathscr{E}^2/120\pi$ watts

$$A = P_r/P_0 = 3\lambda^2/8\pi = 0.1193\lambda^2.$$

This would apply to a Hertzian oscillator with such large terminal capacitances that the current is approximately uniform over its length.

In the case of a half-wave dipole the current is distributed sinusoidally over its length and since its effective length is therefore $2l/\pi$

$$I = \frac{2}{\pi} \cdot \frac{\mathscr{E}l}{2R_{rad}} \text{ and } P_r = \frac{\mathscr{E}^2l^2}{\pi^2R_{rad}} \text{ watts.}$$

In this case $R_{rad} = 73.2$ ohms (see Editorial April 1945) and $l = \lambda/2$,

therefore $$P_r = \frac{\mathscr{E}^2\lambda^2}{4\pi^2 \times 73.2}$$

and $$A = P_r/P_0 = \frac{120\lambda^2}{4\pi \times 73.2} = 0.1305\lambda^2.$$

It is interesting to note that this effective area of a half-wave dipole is a little more than that of a rectangle of the same length as the aerial with a width equal to half the length. It is the power transmitted through this rectangle that the aerial abstracts and delivers to the load resistance. It will be noticed that the effective area of the Hertzian oscillator for the same wavelength is only slightly less, although its length may be very much less, but to obtain the assumed uniformity of current, large terminal capacitance is essential and we may picture the aerial as a short wire joining the centres of two large parallel plates. Hence the effective area gains in width what it loses in length.

Isotropic Aerial

This is the name given by the author to a hypothetical aerial which has the same radiation intensity in all directions. It will have an effective area equal to two-thirds of that of the small dipole, for if we consider a spherical shell around the transmitter, the energy per cm^3 will be the same everywhere if the transmitter is isotropic, and the energy in any narrow zone will be proportional to the volume of the zone, that is, to $\sin\theta$ where θ is the angle to the polar axis, whereas, if the transmitter is an ordinary small dipole the energy density is proportional to $\sin^2\theta$, and therefore the energy in any narrow zone will be proportional to $\sin^3\theta$. On integrating from 0 to π to find the total energy, $\sin\theta$ gives 2 and $\sin^3\theta$ 4/3; hence for the same current and the same radiation in the equatorial plane the isotropic aerial radiates 1.5 times as much power as the dipole. Its radiation resistance is therefore 1.5 times as great and its received power for a given P_0 only two-thirds of the received power of the dipole; its effective area is therefore two-thirds of that of the dipole.

We have seen that for a short dipole $A = 3\lambda^2/8\pi$; hence for the isotropic aerial $A = \lambda^2/4\pi$. If such an isotropic aerial were radiating a power P_t, the value of P_0 at a distance d would be $P_t/4\pi d^2$ which we see is equal to $P_tA/d^2\lambda^2$, and if A_t and A_r are the respective effective areas of the sending and receiving aerials, we have

$$P_r/P_t = A_rA_t/d^2\lambda^2$$

Any change in the character of either aerial causes an appropriate change in its effective area which must be known before the above formula can be used.

Broadside Arrays

A broadside array of n aerials each $\lambda/2$ long and $\lambda/2$ apart with a reflector that doubles the gain is found to have an effective area of $n \times \lambda/2 \times \lambda/2$ that is, about equal to the actual area. This is stated to agree with calculations published by Pistolkors in *Proc. I.R.E.* in 1929.

Experiment shows that aerials with parabolic reflectors have an effective area equal to about two-thirds of the projected area of the reflector; for a long horn it was found to be 81 per cent, and for an optimum horn, designed to give maximum gain, 50 per cent of the aperture area.

In conclusion, the author says "It is suggested that radio engineers hereafter give the radiation from a transmitting aerial in terms of the power flow per unit area, which is equal to P_tA_t/λ^2d^2, instead of giving the field strength in volts per metre. It is also suggested that an antenna be characterized by its effective area instead of by its power gain or radiation resistance. The ratio of the effective area to the actual area of the aperture of an antenna is also of importance in antenna design, since it gives an indication of how efficiently the antenna is utilizing the physical space it occupies."

With regard to the first suggestion, since $P_0 = \mathscr{E}^2/120\pi$ it seems immaterial whether one uses the power flow P_0 per unit area or the field strength $\mathscr{E}$, but if one uses P_0 then the effective area is obviously more convenient than the radiation resistance; if, however, one is given the field strength $\mathscr{E}$ then the radiation resistance is the more convenient characteristic. "Power gain" seems a vague and unsatisfactory term. If an aerial has an extensive reflector such as a horn or parabola the area of the aperture is quite definite, but in many cases "the actual area of the aperture of an antenna" can bear little relation to the effective area.

G. W. O. H.

OSCILLATOR POWER RELATIONS*

By R. E. Burgess, B.Sc.

(Communication from the National Physical Laboratory)

SUMMARY.—The amplitude and power relations are derived for a class of valve-maintained oscillators in which the source of power can be represented as a negative conductance element which has a characteristic limited by a term proportional to a higher odd-power of the voltage. The analysis is based on the classical work of E. V. Appleton and B. van der Pol. The coupling conditions for obtaining the maximum output power from such a source are deduced and shown to differ fundamentally from the impedance-match conditions appropriate to linear systems. It is shown that the intrinsic oscillatory-circuit losses are purely parasitic in the transfer of power to an external load-circuit, and there is no question of a resistance match.

The response of such an oscillator circuit to a small external e.m.f. having a frequency different from the oscillation frequency is considered. It is found that the circuit effectively has a positive conductance which is proportional to the excess negative conductance producing oscillation.

1. Introduction

IT is frequently required to couple a load to a valve-maintained oscillator and to know how the power obtained will depend upon the coupling, and in particular how to obtain the maximum power in the load. In such problems it is important to remember that the valve-maintained oscillator is a non-linear source of power, and that the usual linear-network theorems (e.g., Superposition or Thévenin's) are inapplicable. Derived quantities such as impedance and admittance will, in such cases, depend upon arbitrary definition, and it is therefore preferable to describe the behaviour of such systems in terms of directly observable quantities such as current, voltage and power.

The present note illustrates this by reference to a class of oscillators in which the non-linear element which both maintains and limits the oscillation is representable as a negative conductance which can be expressed as a simple binomial function of voltage. The analysis is based on the classical work of Appleton and van der Pol[1, 2, 3, 4, 5] and its development is limited to power relationships, and their significance in practical problems.

2. Amplitude Relations

The circuit considered is shown symbolically in Fig. 1, in which L, C and G represent the oscillatory circuit and N the negative conductance element which maintains oscillation. It is assumed that the current-voltage characteristic of N has the form

$$i = -av + bv^{2n+1} \quad .. \quad .. \quad .. \quad (1)$$

where the first term represents the main component of the falling characteristic and the second term the essential non-linearity which limits the amplitude of oscillation. This type of characteristic is a sufficiently close representation of a wide range of actual cases to enable the essential features of their behaviour to be deduced. The omission of even-power terms is without effect on the

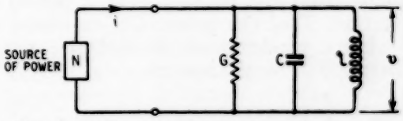

Fig. 1. *Schematic circuit of valve oscillator.*

amplitude while the inclusion of only one odd-power term avoids the possibility of hysteresis and the existence of more than one stable amplitude[3]. Fig. 2 illustrates the characteristics for $n = 1$, 4 and ∞. The latter case may be termed " ideal " and is a limiting form not practically realizable.

The turning values v_1 of v are given by

$$v_1 = \pm \left(\frac{1}{2n+1} \cdot \frac{a}{b} \right)^{1/2n} \quad .. \quad .. \quad (2)$$

and are of particular importance. In terms of v_1, the characteristic becomes :—

$$i = -av \left\{ 1 - \frac{1}{2n+1} \cdot \left(\frac{v}{v_1} \right)^{2n} \right\} \quad .. \quad (3)$$

The range $-v_1$ to $+v_1$ is that for which N has a negative conductance. The conductance is conveniently expressible as

$$g = \frac{di}{dv} = -a \left[1 - \left(\frac{v}{v_1} \right)^{2n} \right] \quad .. \quad (4)$$

* MS accepted by the Editor, February 1946.

For the calculation of the amplitude and power relationships it will be assumed that the potential difference v across the oscillatory circuit is a pure sine wave

$$v = A \sin \omega_0 t \quad .. \quad .. \quad .. \quad (5)$$

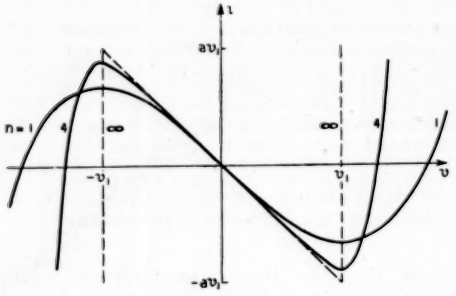

Fig. 2. *Typical current-voltage characteristic of N* $(i = -av + bv^{2n+1})$

Small departures from this wave form will not materially affect the amplitude and power relationship with which we are now concerned.

The amplitude of oscillation, A, can be deduced from the balance over a period of oscillation, T of the power Gv^2 expended in the circuit conductance G and the power $-iv$ supplied by the source.

$$O = \int_0^T (Gv^2 + iv)dt = \frac{A^2T}{2}(G - a + A^{2n}b\,F_n)$$

where

$$F_n = \frac{1}{\pi}\int_0^{2\pi} \sin^{(2n+2)}\theta\, d\theta = \frac{3.5\ .\ .\ .\ (2n+1)}{4.6\ .\ .\ .\ (2n+2)} \quad .. \quad .. \quad (6)$$

is a numerical factor, less than unity, which depends upon the characteristic of the negative conductance element.

Thus,

$$A = \left(\frac{a-G}{bF_n}\right)^{1/2n} \quad .. \quad .. \quad .. \quad (7)$$

or, in terms of v_1

$$A = v_1\left(\frac{2n+1}{F_n}\ .\ \frac{a-G}{a}\right)^{1/2n} \quad .. \quad (8)$$

Now a is the magnitude of the negative conductance N for small oscillations, and for oscillation to occur, a must exceed G, the amplitude depending on the excess negative conductance $(a - G)$. In the limiting case of $G = 0$ the amplitude A has its maximum possible value of

$$A_0 = v_1\left(\frac{2n+1}{F_n}\right)^{1/2n} \quad .. \quad .. \quad (9)$$

which decreases from $2v_1$ at $n = 1$ to $1.44v_1$ at $n = 4$ and tends to v_1 as n tends to infinity. The maximum possible amplitude of oscillation is therefore equal to $2v_1$ which is only attainable with a cubic characteristic and zero circuit losses.

3. Optimum Power Relations

It will now be assumed that the effective conductance of the oscillatory circuit is continuously variable, either by adjustment of the coupling to a load or direct variation of its effective resistance or in any other way. The power dissipated in the circuit is

$$P = \tfrac{1}{2}A^2G = \tfrac{1}{2}v_1^2G\left[\frac{2n+1}{F_n}\ \frac{a-G}{a}\right]^{1/n} \quad .. \quad .. \quad (10)$$

This reaches a maximum value of

$$P_{max} = \tfrac{1}{2}v_1^2a\ .\ H_n \quad .. \quad .. \quad (11)$$

where

$$H_n = \frac{n}{n+1}\left(\frac{2n+1}{n+1}\cdot\frac{1}{F_n}\right)^{1/n} = \frac{4n}{n+1}\left(\frac{n\,!^2}{2n\,!}\right)^{1/n} \quad .. \quad .. \quad (12)$$

when the circuit conductance G has its optimum value

$$G_{opt} = \frac{n}{n+1}\ .\ a \quad .. \quad .. \quad .. \quad (13)$$

As a function of n, H_n varies as shown in the following table :—

n	1	2	3	4	5	6	---	∞
H_n	1	1.089	1.105	1.106	1.103	1.099	---	1

that is to say, it is a slowly varying quantity which differs very little from unity. *Therefore the maximum power available from the negative conductance is very approximately equal to that which would be expended in a resistance* $1/a$ *by a sinusoidal oscillation of amplitude* v_1.

Hence the slope of the characteristic of N at the origin and the range of voltage over which the slope is negative are the significant parameters in determining the available power of the element.

4. Power Transfer to a Load

In many cases the oscillatory circuit will be used to supply power to an external load. Such cases, however, do not need any separate analysis, for the above discussion of the optimum power relationships is quite independent of the constitution of the conductance G, which may be wholly due to the intrinsic resistance of the oscillatory circuit, or may be chiefly due to the coupling of the oscillatory circuit to its load. Since, in practice, the useful power is that delivered to the external load, the conductance of the oscillatory circuit itself can be taken into account by subtracting it from the linear term "a" of the negative conductance of the source of power. If "a" is reduced in this way, the relations of the preceding section will be applicable to the transfer of power to an external load G.

The important thing to note is that there is no question of matching the tuned circuit to the load as is sometimes wrongly assumed. The tuned circuit is necessary in order to provide the desired sine-wave oscillation and to determine the frequency but it is an agent as far as the power transfer is concerned, and the loss involved is purely parasitic.

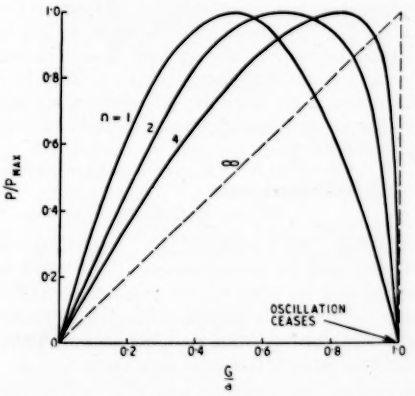

Fig. 3. *Relation between power and load.*

The system is comparable with the transfer of power from a linear source of power to a load by way of a tuned circuit, the analysis of which has been given by Colebrook.[6].

5. Variation of Power and Amplitude with Load

From equations (10) and (11) we can express P/P_{max} as a function of G/a, i.e.,

$$\frac{P}{P_{max}} = \frac{n+1}{n} \cdot \frac{G}{a}\left[(n+1)\left(1-\frac{G}{a}\right)\right]^{1/n} \quad .. \quad .. \quad (14)$$

This is shown in Fig. 3 for $n = 1$, 2, 4 and ∞. It will be seen that for the ideal negative conductance ($n \to \infty$), the power increases linearly with G, since the amplitude remains constant up to $G = a$, at which oscillation will cease abruptly.

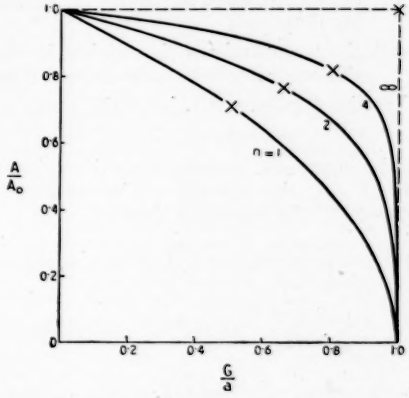

Fig. 4. *Dependence of amplitude on load. Crosses denote load for maximum load power.*

It is noted that the dependence of power on load is more critical than in the case of a linear source of power obeying Thévenin's Theorem and this is due to the reduction of amplitude (eventually to zero) as the loading is increased.

From equation (8), the variation of the amplitude of oscillation with the loading G is given by

$$\frac{A}{A_0} = \left(1-\frac{G}{a}\right)^{1/2n} \quad .. \quad .. \quad (15)$$

where A_0 is the amplitude corresponding to $G = 0$. This relation is shown in Fig. 4 and it will be seen that as n increases, the amplitude tends to remain more constant up to the point at which oscillation ceases.

The dependence of the amplitude and power on the load emphasizes the essential differences between a linear and a non-linear source of power. Clearly one cannot ascribe an effective resistance to an oscillator without introducing an arbitrary definition or method of measurement upon which the apparent value will depend. For this reason it is desirable to consider such non-linear systems in terms of observable quantities, viz., current, voltage or power.

6. Response of the Oscillator to an External E.M.F.

The response of an oscillator to a small external e.m.f. of frequency different from the oscillation frequency is of interest in a number of connections. For example:

(i) the influence of the inherent thermal and shot noise in the oscillator on its output voltage;

(ii) the reception of signals by an oscillating detector;

(iii) the magnitude of the sidebands when the oscillator is amplitude-modulated.

Let the applied e.m.f. have the form

$$e = E \sin \omega t$$

and be impressed in series with the inductive arm L of the circuit. The differential equation of the system is then

$$L\frac{d}{dt}\left[i + Gv + C\frac{dv}{dt}\right] + v = E \sin \omega t$$

or $$v'' + \frac{Gv' + i'}{C} + \omega_0^2 v = \omega_0^2 E \sin \omega t \quad .. \quad .. \quad (16)$$

where $\omega_0^2 = 1/LC$ and the primes denote time differentials.

The resultant voltage across the oscillatory circuit has the form appropriate to the sum of a self-maintained oscillation and a forced vibration, the impedance of the circuit being taken as negligible at harmonic and heterodyne frequencies:

$$v = A \sin \omega_0 t + U \sin (\omega t + \phi) \quad (17)$$

It will be assumed that the applied e.m.f. is small so that U is small compared with A, and it is found on substituting for v in equation (16) and expanding as far as U^2/A^2, that the amplitude of the self-oscillation of angular frequency is given by

$$A \approx \left(\frac{a-G}{bF_n}\right)^{1/2n}\left[1 + n(n+1)\frac{U^2}{A^2}\right]^{-1/2n}$$

$$\approx \left[A_1^2 - (n+1)U^2\right]^{1/2} \quad .. \quad (18)$$

where $A_1 = \left(\frac{a-G}{bF_n}\right)^{1/2n}$ is the amplitude in the absence of an external e.m.f. (Equation 7).

The amplitude of the forced vibration of angular frequency ω is given by

$$U = \frac{E}{\sqrt{(1-\omega^2 LC)^2 + \omega^2 L^2 \{n(a-G)\}^2}} \quad (19)$$

which is the form appropriate to a linear circuit having the constants L, C and G' where the effective conductance G' is given by

$$G' = n(a - G) \quad .. \quad .. \quad .. \quad (20)$$

which may be greater or less than the true circuit conductance G.

The effective bandwidth of the system between half-power points for an external e.m.f. is thus

$$B = \frac{G'}{2\pi C} = \frac{n(a-G)}{2\pi C} = \frac{f_0}{Q} n\left(\frac{a}{G} - 1\right) \quad .. \quad .. \quad (21)$$

where $Q = \frac{\omega_0 C}{G}$ is the magnification of the oscillatory circuit by itself.

As might be expected G' is equal to the sum of the circuit conductance G and the average value of the conductance of N over a cycle of oscillation:

$$G + \bar{g}$$
$$= G + \frac{1}{2\pi}\int_0^{2\pi}\left[-a + (2n+1)b(A \sin \theta)^{2n}\right] . d\theta$$
$$= G + \left[-a + (n+1)(a-G)\right]$$
$$= n(a-G) \quad .. \quad .. \quad (22)$$

The effective magnification U/E and hence the selectivity of the circuit for a small applied voltage of frequency different from the oscillation frequency is seen to be the greater the nearer the system is to the edge of oscillation (i.e. the smaller $a-G$ is). If the excitation (a/G) is great, however, the circuit is apparently heavily damped and will have low selectivity.

7. Acknowledgements

The author is indebted to Mr. F. M. Colebrook for suggestions regarding the problem and its presentation. The work was carried out as part of the programme of the Radio Research Board and this paper is published by permission of the Department of Scientific and Industrial Research.

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SIGNAL-NOISE RATIO AT V.H.F.*

By M. J. O. Strutt and A. van der Ziel

(Natuurkundig Laboratorium der N. V. Philips' Gloeilampenfabrieken.)

SUMMARY.—In Section 1 the two basic concepts of correlation between fluctuation currents or voltages and of the decomposition of fluctuating quantities into singly periodic components are discussed briefly. Section 2 considers the different sources of noise at the input of a triode used in an amplifier circuit. The fluctuation voltage as well as the signal voltage between grid and cathode are evaluated. Thereupon the corresponding signal current and fluctuation current in the short-circuit connection between anode and cathode are given. By division of the latter by the former current the noise ratio is found in Section 3. This noise ratio consists of two components. The one due to the presence of the valve may be minimized by a proper choice of aerial coupling and of detuning (if necessary) of the input circuit. The most favourable values of these two quantities are calculated. The minimum value of the valve's contribution to the noise ratio thus obtained is shown in a set of curves as dependent on two parameters. These results are discussed in Section 4. An experimental example is given, showing the reduction of the noise ratio by detuning the input circuit. This experimental evidence is shown to confirm the theoretical results.

Section 5 deals with a grounded-grid stage. Upon proper handling the equations of the grounded-cathode case are shown to be valid with irrelevant changes of symbols. Hence the chief results are found to be also dependent on the same set of curves. A considerable possible decrease of noise ratio is conjectured. A velocity-modulation valve is considered in Section 6 instead of the density-modulation triode in the preceding sections. Here, again, the discussion and equations of Section 3 may be directly applied with slight changes. Due to the special conditions of the present case the minimum noise ratio turns out to be rather high. A special device by which it might be brought down considerably is indicated. Section 7 deals with the requirements as to valve construction resulting from the preceding discussion. The main points are: uniform electron paths and speeds in the valve and low dielectric and other losses at the input electrodes. It is shown that the proposed reduction of noise ratio applies also to wide-band reception (e.g. 10 Mc/s band-width). In order to compensate the loss of gain incurred by the reduction of noise ratio suitable feedback may be applied. The proposed reduction is not confined to amplifier stages but may also be applied to mixer stages. In the Appendix the interrelation of the noise figures introduced by K. Fraenz, D. O. North, H. T. Friis and the present noise ratio are discussed.

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1. Introduction.
2. Noise ratio of a grounded-cathode amplifier.
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4. Discussion and experimental evidence.
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6. Velocity-modulation valves.
7. Valve construction and stage design.
8. Appendix: noise ratio and noise figures.

1. Introduction

In ultra-high frequency reception the signal-to-noise ratio is often much lower than with similar signals at lower frequencies. On the other hand, man-made noise and noise from extra-terrestrial sources may be less. Hence an increase of the said ratio by proper stage design and valve construction seems especially appropriate in the lower meter-wave and in the decimeter-wave bands.

Several definitions of noise figures have been published since 1941 (references [3, 4, 5, 6, 8, 10, 13, 18]). It seems, however, that some discussion is still needed before the universal adoption of one particular figure is secured. Therefore, no explicit use will be made of any of these definitions in this article. Instead, we shall make use of a noise ratio, defined in Section 2.

Two concepts are fundamental to a full understanding of the present discussion. The first is the concept of the correlation of spontaneous fluctuations. Two spontaneous fluctuation currents flowing through the same conductor are completely correlated, if the resulting r.m.s. fluctuation current is obtained by adding the individual r.m.s. current values. They are incorrelated if the mean-square value of the resulting current is obtained by adding the mean-square values of the individual currents. We do not need the definition of partly correlated currents.

The second is the concept of the decomposition of spontaneous fluctuation currents into an infinite sum of single harmonic components. The latter represent simple alternating currents and we may use them in the calculations as such. If the frequency-band within which the spontaneous fluctuation currents

* MS. accepted by the Editor, January, 1946.

are considered is sufficiently narrow compared to its average frequency value, the current in question behaves as a singly periodic alternating current of fluctuating amplitude.

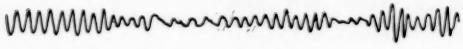

Fig. 1. *Fluctuating current corresponding to a frequency band Δf centred round a frequency f_o such that $f_o \gg \Delta f$. In this case $f_o \simeq 14\,\Delta f$, and the amplitude shows random fluctuations of average period approximately equal to $1/\Delta f$.*

The average period of fluctuation is approximately equal to the reciprocal value of the said frequency-band. Perhaps Fig. 1, showing the experimental representation of such a fluctuating current may be helpful to visualize its action in a complex network, since it is similar to that of an ordinary alternating current.*

2. Noise Ratio of a Grounded-cathode Amplifier

Signal currents and voltages will be denoted by I and V respectively with suffixes to indicate different individual values. Fluctuation currents and voltages in the above sense will be denoted by i and v respectively, again using suffixes to indicate different values. The signal at the amplifier input may be supposed to be due to a receiving aerial of effective resistance R_a at its terminals. A triode will be used in the stage under consideration in order to avoid unessential complications attending the use of tetrodes or pentodes (the latter are dealt with in Section 4). The cathode is supposed to be the electrode which is common to input and output, thus forming a so-called "grounded-cathode" amplifier stage. The anode will be assumed to be connected to the cathode by a lead of very small impedance compared to the internal valve impedance between anode and cathode. We shall evaluate the signal current I and the fluctuation current i flowing in this short-circuit connection. The ratio $\overline{i^2}/I^2$ is called the *noise-ratio* of the amplifier stage. The horizontal dash above i^2 denotes its average (arithmetic mean) value, taken either by considering a lapse of time, very long compared with the main periods involved in i, or by considering a great number of identical amplifiers and averaging the values of i^2 obtained. This noise ratio is related by simple formulæ to the noise-figures proposed in previous publications. We need not, however, go into these relations here (see Appendix).

Coupling between the signal source, e.g. the aerial output, and the input tuned-circuit is represented symbolically by a transformation ratio n, stepping up the source-resistance R_a to n^2R_a in parallel with the impedance of the said tuned-circuit. The signal source is also the seat of spontaneous fluctuations. In series with the above resistance R_a two constant-voltage generators of zero internal impedance are assumed, one supplying the signal voltage V_a and the second the fluctuation voltage v_a. The latter corresponds to the small frequency band Δf centred round the signal frequency and may be assumed to be due to thermal electronic fluctuations in the resistance R_a at a temperature T_a, thus[1,14,17]: $\overline{v_a^2} = 4kT_aR_a\Delta f$ where k is Boltzmann's constant ($1.38.10^{-23}$ joule per degree Kelvin). Instead of the said constant-voltage generators it is simpler to make use of constant-current generators of infinite internal impedance supplying signal and signal-source fluctuation currents to the valve's input circuit.

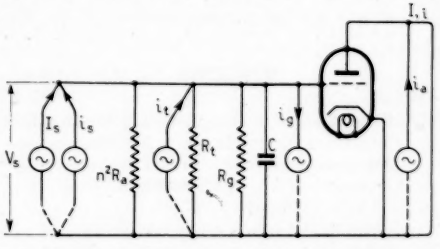

Fig. 2. *Input circuit connected to cathode and grid of a triode showing the transformed signal-source resistance n^2R_a, the tuned circuit's resonant resistance R_t and the valve's input resistance R_g due to electron and to inductive effects. The constant-current generators supply the signal current I_s, the source's fluctuation current i_s, the tuned circuit's fluctuation current i_t, the induced grid fluctuation current i_g and the shot-effect fluctuation current i_a. The signal current I and resulting fluctuation current i in the short-circuit anode lead are calculated.*

Taking into account the transformation ratio mentioned above, we obtain a current supplied by the constant-current signal generator:

$$I_s = \frac{nV_a}{n^2R_a} = \frac{V_a}{nR_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

* Due to Kappler, see *Annalen der Physik*, Vol. 2, 1931, p. 233.

The constant-current noise generator supplies the current

$$\overline{i_s^2} = \frac{\overline{v_a^2}}{n^2 R_a^2} = \frac{4kT_a R_a \Delta f}{n^2 R_a^2} = \frac{4kT_a \Delta f}{n^2 R_a} \qquad (2)$$

The complete input-circuit is shown in Fig. 2. Parallel to the transformed source-resistance n^2R_a is the resonant impedance R_t of the tuned circuit, the input resistance R_g between grid and cathode of the valve due exclusively to electronic causes or to lead inductances and a capacitance C indicating a detuning (if desired) of the total input circuit.

Two sources of noise, besides the one already mentioned, have still to be taken into account. The first one is the thermal noise of the resistance R_t at room temperature T. It may be represented by a constant-current generator supplying the fluctuation current

$$\overline{i_t^2} = \frac{4kT\Delta f}{R_t} \qquad .. \quad .. \quad .. \quad (3)$$

The second one is due to the induced fluctuation current at the grid[6,12,16], and will be indicated by i_g. It may be described as being due to thermal electronic fluctuations in a resistance R_g at a temperature T_g, thus[12]

$$\overline{i_g^2} = \frac{4kT_g \Delta f}{R_g} \qquad .. \quad .. \quad .. \quad (4)$$

Due to the said current generators a signal voltage V_s and a noise voltage v_s arise between grid and cathode at an angular frequency ω

$$\left.\begin{aligned} V_s &= \frac{I_s}{\frac{1}{n^2R_a} + \frac{1}{R_t} + \frac{1}{R_g} + j\omega C} \\ \overline{v_s^2} &= \overline{\left| \frac{i_s + i_t - i_g}{\frac{1}{n^2R_a} + \frac{1}{R_t} + \frac{1}{R_g} + j\omega C} \right|^2} \end{aligned}\right\} \qquad (5)$$

The vertical dashes in the latter expression indicate its modulus whereas the horizontal dash denotes averaging, taking due account of the non-correlation between i_s, i_t and i_g. What we are interested in are the signal and the noise currents in the short-circuit anode lead due to the above voltages. Denoting the transadmittance of the valve from grid to anode by S we obtain an anode signal current $I = SV_s$. As to the noise current i in the anode lead, this consists firstly of Sv_s and secondly, of the noise current i_a circulating in this lead if the grid is connected to the cathode across a low impedance while maintaining the previous d.c. operating conditions. Thus

$$\overline{i^2} = \overline{\left| \frac{S(i_s + i_t - i_g)}{\frac{1}{n^2R_a} + \frac{1}{R_t} + \frac{1}{R_g} + j\omega C} + i_a \right|^2} \qquad .. \quad .. \quad (6)$$

The noise ratio is obtained by division of Equ. (6) by $|SV_s|^2$, wherein V_s is given by Equs. (5) and (1).

3. Conditions for Minimum Noise Ratio

In studying the conditions for a minimum noise-to-signal ratio two outstanding facts have to be borne in mind. Firstly, the complete correlation of i_g and i_a in an ideal triode, both fluctuations being due to the same electron stream through the valve. Secondly, the phase-angles of the transadmittance S and of i_g relative to i_a due to electron transit times. We may express these angles thus[18]

$$\left.\begin{aligned} S &= |S|\, e^{-j\phi_o} \\ i_a &= |i_a|\, e^{-j\phi_a} \\ i_g &= j\,|i_g|\, e^{-j\phi_g} \end{aligned}\right\} \qquad j = +\sqrt{-1} \qquad .. \quad .. \quad (7)$$

Only the relative phase-angle $\phi_g - \phi_a$ of the correlated currents i_g and i_a is relevant.

The modulus $|S|$ of the transadmittance does not vary appreciably from low to extreme-high frequencies[19]. At a frequency of 600 Mc/s, ϕ_0 may be 120° in a triode of the usual electrode-dimensions. The modulus of i_a also does not vary appreciably with frequency up to the said value in many cases. Finally, if the ratios R_t/R_g and R_t/n^2R_a are increased by increasing the quality-figure of the tuned circuit until they become large compared to unity, the expressions $1/R_t$ and i_t may be dropped out of Equ. (6). We shall assume that this condition is satisfied. Inserting Equ. (7) we thus obtain a noise ratio :

$$\left.\begin{aligned} \frac{\overline{i^2}}{I^2} &= \frac{\frac{\overline{v_a^2}}{R_a^2} + \overline{i_g^2}\frac{R_g}{R_a}w}{V_a^2/R_a^2} \\ w &= \left| 1 - \left| \frac{i_a}{i_g S R_g} \right| \frac{1}{j}\, e^{j\phi} \left(\frac{R_g}{n^2 R_a} + 1 + j\omega C R_g \right) \right|^2 \times \frac{n^2 R_a}{R_g} \\ \phi &= \phi_o + \phi_g - \phi_a \end{aligned}\right\} \qquad .. \quad .. \quad (8)$$

A minimum noise ratio obviously coincides

with a minimum value* of w. Introducing the symbols

$$\overline{\left|\frac{i_a}{i_g S R_g}\right|^2} = a^2, \quad \frac{R_g}{n^2 R_a} = x, \quad \omega C R_g = y, \qquad (9)$$

the expression for w becomes

$$w = 1/x \times [1 - a\{(x+1)\sin\phi + y\cos\phi\}]^2 + a^2/x \times \{(x+1)\cos\phi - y\sin\phi\}^2 \qquad (10)$$

The values of x (i.e., the coupling ratio n) and of y (i.e., the detuning) may now be chosen so as to minimise w. Varying y a minimum is reached if

$$ay = \cos\phi, \qquad (11)$$

this minimum value being

$$\mathrm{Min}_y\,(w) = \frac{\{a(x+1) - \sin\phi\}^2}{x}$$

Varying x, this expression attains a minimum value, if:

$$a^2x^2 = (a - \sin\phi)^2 \qquad (12)$$

The condition (11) fixes the detuning capacitance. This may be positive if $\cos\phi$ is positive and negative (i.e., inductive detuning) if $\cos\phi$ is negative. As to the latter condition (12) we shall assume a unequal to $\sin\phi$. For if these values were equal we would obtain $x = 0$ and hence no signal would result at the valve input ($I_s = 0$). Two cases corresponding to Equ. (12) have still to be considered: firstly $(a - \sin\phi) > 0$ and, secondly, $(a - \sin\phi) < 0$. In the first case, the minimum value of w is

$$\mathrm{Min}_{xy}\,(w) = 4a\,(a - \sin\phi) \qquad (13)$$

and hence is positive, while in the second case it is zero. Theoretically this second case is extremely interesting, as the contributions of the valve would then completely disappear from the noise ratio. This would hence be equal to the noise ratio at the terminals of the signal source (e.g., receiving antenna). From a practical point of view it is not so important if w disappears completely. The only important condition is that $\overline{i_g^2}w$ should be small compared to $\overline{i_s^2}$ in Equ. (8). In words:—the contribution of the valve to the noise ratio should be negligible.

We shall now deal with the case when no detuning is applied. Hence $y = 0$ and the minimum condition for w becomes

$$a^2x^2 = 1 - 2a\sin\phi + a^2 \qquad (14)$$

* See Appendix for further definition of w.

the minimum value of w being

$$w_{\min} = 2a(\sqrt{1 - 2a\sin\phi + a^2} + a - \sin\phi) \qquad (15)$$

When $\sin\phi = 1$ and $a \leqslant 1$ this minimum value becomes zero. Comparing Equs. (14) and (12) it is seen that x^2 is larger and hence n^2 smaller in the former case. Thus, optimal coupling is weaker if detuning is applied, and especially so, if $\sin\phi$ is small.

4. Discussion and Experimental Evidence

The reduction and eventually more or less complete compensation of the valve's contribution to the noise ratio is due to a fluctuation voltage component, completely correlated to i_g, arising across the input circuit. By proper detuning this voltage causes an anode fluctuation current component, due to the valve's transadmittance, which is counterphase (phase-difference of π radians) with respect to i_a. Hence the correlation

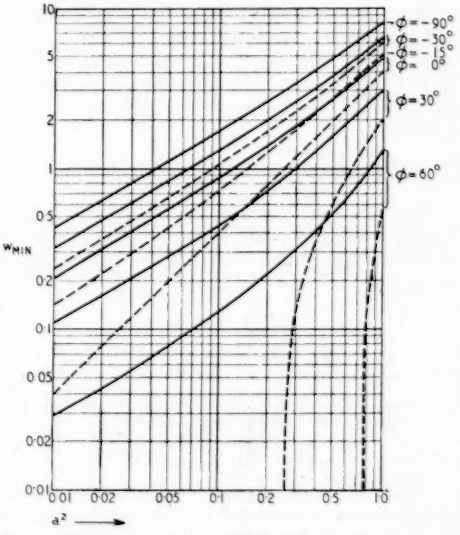

Fig. 3. *The minimum value of the expression w (vertical scale) of Equs. (8) and (10) varying x and y of Equ. (9) as dependent on a^2 of Equs. (9) and on ϕ of Equ. (8). Full curves without, and dotted curves with, optimal detuning.*

of i_g to i_a is essential. In an ideal plane triode this correlation exists to a fairly complete degree.

With the aid of the so-called equivalent noise-resistance R_n of the valve [6], defined by:

$$\overline{i_a^2} = 4kT\,|S^2|\,R_n\Delta f, \qquad (16)$$

T being again the room temperature (e.g., 203 degrees Kelvin), the expression (9) for a^2, using Equ. (4) may be written as:

$$a^2 = \frac{R_n}{R_g}\,\frac{T}{T_g} \qquad (17)$$

By applying Equs. (13) and (15) the minimum value of w may be calculated as dependent on a^2 for different values of ϕ with or without detuning. This is shown in Fig. 3. It appears from these curves that detuning has relatively little effect at negative values of ϕ. As the noise ratio of Equ. (8) may be written as[18]

$$\frac{\overline{i^2}}{I^2} = \frac{kT\Delta f\,(T_a/T + wT_g/T)}{V_a^2/4R_a} \qquad (18)$$

by using Equs. (2) and (4), this Fig. 3 is directly suited to the discussion of the relative importance of the two contributions to this ratio, corresponding to the two terms between brackets in the numerator of (18). The ratio of T_a/T is often larger than 1, e.g. of the order of 2 or more, whilst T_g/T is usually, with oxide-coated cathodes, of the order of 5. Assuming 2 and 5 for these ratios, we would obtain the result that w should be below, say, 0.2 in order to make a further decrease of w relatively uninteresting. From Fig. 3 we may see, if ϕ and a^2 are given, whether this may be realized. As an example, we assume a^2 may be 0.5 at 1 metre wavelength. Obviously, by Fig. 3, w cannot attain values below 0.2 at this value of a^2 if ϕ is zero or negative with or without detuning. If, however, ϕ could be, say, 60° positive, such values could easily be obtained.

If a tetrode or a pentode is used instead of a triode, the fluctuation current i_a consists of two parts, one of which is completely correlated to i_g, the other being, however completely uncorrelated to i_g[6,16]. Hence, the reduction of the noise ratio is mainly confined to the first part of i_a in this case. Denoting the said decomposition of i_a by

$$\overline{i_a^2} = \overline{i_{a_1}^2} + \overline{i_{a_2}^2} \qquad (19)$$

the reduction of w obtainable under ideal conditions, could roughly not be better than to $\overline{i_{a_2}^2}/\overline{i_a^2}$ multiplied by the original value of w.

The noise ratio of a particular valve, a pentode, used in the first stage of an amplifier[8], was measured at 300 Mc/s. The measured figures represent the expression in brackets of Equ. (18) with $T_a = T$ (see Fig. 4, taken from a paper by W. Kleen[8]). It is seen that a reduction of this figure from about 21 to 17 is obtained by detuning, leaving, however, the coupling unaltered. In the valve under consideration $\overline{i_{a_1}^2}$ accounts for about 40 per cent and $\overline{i_{a_2}^2}$ for about 60 per cent of $\overline{i_a^2}$. Hence the reduction should be less than from the original 21 to about 13. Due to non-optimal coupling at the detuned position and to the figures of Fig. 4 including some additional components besides wT_g/T, this optimal reduction is not obtained. This experiment tends to show, however, that worth-while reductions may actually be effected using suitable valves and circuits. We may also conclude from Figs. 3 and 4 that ϕ is positive in the present case.

5. The Grounded-grid Amplifier Stages

In the above discussion the cathode was assumed to constitute the electrode common to input and output. In recent years, however, amplifier circuits have been discussed, in which the control grid is the common electrode[2,7,9]. The possibilities of reducing the noise ratio in amplifiers using a grounded-grid valve will now be discussed.

The input circuit is exactly the same as that of Fig. 2. Instead of connecting the anode to the cathode by a short-circuit lead it should now be thus connected to the grid (of course, always taking due care of direct current connections). Assuming again a triode, the fluctuation current in the said anode-grid lead, if the input is short-circuited, is again i_a. But the fluctuation current i_g to the grid must now be replaced[16] by $i_a + i_g = i_c$. This current i_c is completely correlated to i_g in an ideal triode, as is i_a. With these changes we obtain instead of Equ. (8) the noise ratio

$$\left.\begin{aligned} \frac{\overline{i^2}}{I^2} &= \frac{\overline{v_a^2}/R_a^2 + \overline{i_c^2}\,\frac{R_g}{R_a}u}{V_a^2/R_a^2} \\ u &= \left|1 - \left|\frac{i_a}{i_c SR_g}\right| \frac{e^{j\psi}}{j}\left(\frac{R_g}{n^2R_a} + 1 + j\omega CR_g\right)\right|^2 \times \frac{n^2R_a}{R_g} \\ \psi &= \phi_0 - \phi_a - \pi/2 \qquad i_c = |i_c| \end{aligned}\right\} \qquad (20)$$

The value of R_g in this case is different from the grounded-cathode case. At r.f.,

R_g is approximately equal to $1/S$ in the present case.

This ratio attains a minimum value if u is as small as possible. Using the notations:

$$\left.\begin{aligned} \overline{i_c^2} &= 4kT\,|S^2|\,R_c\Delta f, \\ b^2 &= \left|\frac{i_a}{i_c S R_g}\right|^2 = \frac{R_n}{|S^2|R_g^2 R_c} \\ x &= \frac{R_g}{n^2 R_a} \qquad y = \omega C R_g \end{aligned}\right\} \quad .. \quad (21)$$

the expression for u becomes:

$$u = 1/x \times [1 - b\{(x+1)\sin\psi + y\cos\psi\}]^2 + \\ + b^2/x \times \{(x+1)\cos\psi - y\sin\psi\}^2 \quad .. \quad .. \quad (22)$$

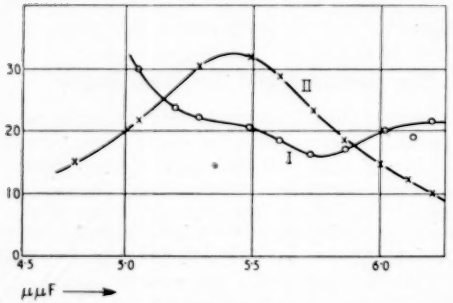

Fig. 4. *Experimental curve representing the expression between brackets in the numerator of Equ. (18) with $T_a = T$ (curve I, vertical scale) as dependent on the input circuit's tuning capacitance (horizontal scale) and at constant coupling, optimal for noise ratio at the tuned position. Curve II represents the input signal voltage squared (vertical linear scale) as dependent on tuning capacitance, and hence affords a check on the tuning position.*

Comparing this Equ. (22) with Equ. (10) we see that the procedure used in dealing with the latter may be repeated in the present case. Hence the resulting minimum conditions and values of u may be taken from Equs. (11) to (15), replacing w by u, a by b and ϕ by ψ. The curves of Fig. 3 are also valid in the present case, replacing a by b and ϕ by ψ. Going by the experimental evidence of ϕ being positive and small in the grounded-cathode case corresponding to Fig. 4, we might conjecture ψ to be near $-\pi/2$ in the present case at not too high frequencies. This would mean nearly zero optimal detuning by the equivalent of Equ. (11). A considerable reduction of the noise ratio may be expected also in the present case by the proposed coupling and (if necessary) detuning.

6. Velocity Modulation Valves

We consider a velocity-modulation valve [15] with two adjacent input electrodes and two similar output electrodes, both pairs being passed by the electron stream. The spontaneous fluctuations in density of this stream at a particular point of its path may be due to initial density fluctuations at a previous point or to spontaneous velocity fluctuations at such point, transferring to density fluctuations along the path. With the usual operational data it may be shown that the latter effect is in most cases unimportant.

The initial density fluctuations will decrease along the path due to velocity-distribution of electrons in the stream. This effect, too, may be shown to be usually negligible. Hence in dealing with spontaneous fluctuations we may confine our attention to the effects caused by initial density fluctuations of the stream. The ensuing fluctuation voltages and currents are completely correlated. The output-electrodes are assumed to be connected by a short-circuit lead. We shall evaluate the fluctuation current as well as the signal current in this lead and thus obtain the noise ratio.

The input-circuit coincides with Fig. 2, replacing i_g by i_f, denoting the induced fluctuation-current due to the spontaneous fluctuations of the electron stream passing the input electrodes. The transadmittance from input to output will again be indicated by the complex symbol S. The distance between the two adjacent input electrodes of the velocity-modulation valve as well as between the two adjacent output electrodes will be assumed so short that the product of angular frequency ω and electron transit time between the electrodes of each pair is small compared to unity. With this assumption a simple relation exists between i_f and the spontaneous fluctuations of the electron stream. If I_d denotes the average direct current corresponding to the stream and e the electron charge, we have:

$$\overline{i_f^2} = 2eI_dF^2\Delta f \quad .. \quad .. \quad .. \quad (23)$$

The multiplier F^2 is unity if the stream issues from a cathode operating under saturated conditions and may be smaller than unity if I_d is less than the saturation current of the cathode. Equs. (5) and (6) apply exactly to the present case if i_g is replaced by i_f. The signal current in the short-circuit output lead is $I = SV_s$ and the fluctuation current is:

$$\overline{i^2} = \left| \frac{S(i_s + i_t - i_f)}{\frac{1}{n^2R_a} + \frac{1}{R_t} + \frac{1}{R_g} + j\omega C} + i_f e^{-j\omega t_0} \right|^2 \quad .. \quad .. \quad (24)$$

t_0 indicating the electron transit time along the beam from the input to the output pair of electrodes. Thus Equs. (7) have to be replaced by :

$$\left.\begin{aligned} S &= |S|\, e^{-j\phi_0} \\ i_a &= |i_f|\, e^{-j\phi_a} \qquad \phi_a = \omega t_0 \\ i_f &= |i_f| = j\,|i_f|\, e^{-j\phi_f} \quad \phi_f = \pi/2 \end{aligned}\right\} \quad (25)$$

Inserting this notation into the noise ratio obtained by division of (24) by $|SV_s|^2$ we obtain instead of Equs. (8)

$$\left.\begin{aligned} \frac{\overline{i^2}}{I^2} &= \frac{\frac{\overline{v_a^2}}{R_a^2} + \overline{i_f^2}\frac{R_g}{R_a}\,p}{V_a^2/R_a^2} \\ p &= \left| 1 - \left|\frac{1}{SR_g}\right| \frac{e^{j\phi_1}}{j}\left(\frac{R_g}{n^2R_a} + 1 + j\omega CR_g\right)\right|^2 \times \frac{n^2R_a}{R_g} \\ \phi_1 &= \phi_0 + \phi_f - \phi_a = \phi_0 + \pi/2 - \omega t_0 \end{aligned}\right\} \quad .. \quad .. \quad (26)$$

Thus a similar expression to w of Equ. (8) arises in the present case, as denoted by p above. Introducing the abbreviations

$$c = \left|\frac{1}{SR_g}\right| \qquad x = \frac{R_g}{n^2R_a} \qquad y = \omega CR_g \quad .. \quad .. \quad (27)$$

an expression for p, very similar to Equ. (10), is obtained :

$$p = 1/x \times [1 - c\{(x+1)\sin\phi_1 + y\cos\phi_1\}]^2 + c^2/x \times \{(x+1)\cos\phi_1 - y\sin\phi_1\}^2 \quad .. \quad .. \quad (28)$$

The discussion of this expression in order to calculate its minimum value with x and y as variables is obviously similar to that of Equ. (10) and the results of the latter discussion may be directly applied to the present case. Thus by Equ. (11), the optimal detuning is

$$cy = \cos\phi_1$$

or $$\frac{\omega C}{|S|} = -\sin(\phi_0 - \omega t_0) \quad .. \quad .. \quad (29)$$

whilst optimal coupling becomes by Equ. (12)

$$c^2x^2 = (c - \sin\phi_1)^2$$

or $$\left|\frac{1}{n^4S^2R_a^2}\right| = \left\{\frac{1}{|SR_g|} - \cos(\phi_0 - \omega t_0)\right\}^2 \quad .. \quad .. \quad (30)$$

Now, by the theory of velocity-modulation valves[11,15]

$$S = \tfrac{1}{2}\, j\frac{I_d}{V_d}\,\omega t_0\, e^{-j\omega t_0} = \frac{I_d}{2V_d}\,\omega t_0\, e^{-j(\omega t_0 - \pi/2)} \quad .. \quad .. \quad (31)$$

wherein V_d denotes the steady voltage corresponding to the average electron speed along the path between input and output.

Hence $\phi_1 = \phi_0 + \pi/2 - \omega t_0$
$= \omega t_0 - \pi/2 + \pi/2 - \omega t_0 = 0$,

and $c - \sin\phi_1$ is thus positive. This makes a complete annihilation of p impossible, its minimum value corresponding to the conditions (29) and (30) being

$$\text{Min } (p) = 4c\,(c - \sin\phi_1) = 4c^2 = \frac{4}{|SR_g|^2} \quad .. \quad .. \quad (32)$$

We have not yet discussed the value of R_g to be inserted into the above equation. By the condition imposed above on the electron transit time t_1, between the two input electrodes, we obtain :

$$\frac{1}{R_g} = \frac{I_d}{V_d}\frac{(\omega t_1)^2}{24} \quad .. \quad .. \quad .. \quad (33)$$

Hence $$4c^2 = \frac{4}{|SR_g|^2} = \frac{(\omega t_1)^4}{36\,(\omega t_0)^2}$$

and this value, by the assumption that $\omega t_1 \ll 1$, is surely much smaller than unity. The further discussion may be based on Fig. 3, replacing a^2 by c^2, ϕ by $\phi_1 = 0$ and w by p.

The equivalent of Equ. (18) is in the present case[11]

$$\frac{\overline{i^2}}{I^2} = \frac{kT\Delta f\left(\frac{T_a}{T} + \frac{2eI_dF^2}{kTR_g|S|^2}\right)}{V^2/4R_a} \quad .. \quad (34)$$

The expression between brackets may be regarded as a suitable measure for the relative noise at the output (see Appendix). Whereas this expression is of the order of 20 at 300 Mc/s for a pentode valve according to Fig. 4, it is much higher in the present case. Assuming $|S| = 0.12$ mA/V, $R_a = R_g = 20$ kilohms, $F^2 = 1$ and $T_a = T$ (room temperature) we obtain[11] about 2 500. Thus we may conclude that velocity-modulation valves are not well suited for reception purposes under these conditions. The large noise ratio is obviously due to i_f.

It is possible, however, to devise special measures, based on the same principles,

which could bring the above figure down to, say, below 100. One of these devices consists of an extra pair of electrodes preceding the input electrodes, as viewed in the direction of the electron stream. By connecting a suitable impedance between these electrodes a fluctuation voltage may be generated across the said impedance, causing a fluctuation current in the output lead, and entailing a reduction of the original fluctuation current and hence of the noise ratio. To achieve this the distances between the three pairs of electrodes thus needed, as well as the impedances at the first two pairs, have to be chosen carefully.

7. Valve-construction and Stage Design

The reduction of the noise ratio as set forth in Sections 3 to 6 is hampered if the individual electron paths and motions are unequal. This may be easily seen by going through the argument of Section 3, assuming the valve to consist of two parallel sections with different values of R_g, S, ϕ_a and ϕ_g. Obviously, one minimum value of w will be obtained corresponding to each section and the corresponding minimum conditions as to detuning and coupling will in general not coincide. Seeking a most favourable compromise, we shall still be left with a higher resulting value of w than would correspond to the minimum value for each of the sections. Using Fig. 3, a decision may be reached as to what degree of inequality is still tolerable in this respect without undue rise of the noise ratio. Valve construction should comply with these requirements.

It is assumed in Sections 3 to 6 that the resonant resistance R_t of the tuned circuit may be raised to such a level that the corresponding spontaneous fluctuations of input voltage are relatively unimportant. If a resistance exists between the input electrodes which is not due to electronic causes or to lead inductances, but to losses such as dielectric losses or series resistance of electrode leads, this desirable increase of R_t will be limited. In fact, this resonant resistance, including the said losses, must always remain smaller than the said shunt loss-resistance. Additional non-correlated fluctuations will arise from this cause and entail a rise of noise ratio. This may be accounted for by the addition of a term n^2R_a/R_t to the expression between brackets in the numerator of Equ. (18) and of a term R_g/R_t in the round brackets of w [Equ. (8)]. It is thus possible to judge the value of R_t required to make the contribution of these terms to the over-all noise figure negligible. R_t should be large compared with R_g and this is hard to achieve with velocity-modulation valves.

Strictly, the reduction of noise ratio as proposed above applies only to a frequency band Δf which is very narrow compared to its corresponding average frequency. In what way do our results apply to wide-band reception? An answer to this question under practical conditions may be obtained from Fig. 4. The overall resonant resistance of the input-circuit was less than a thousand ohms at 300 Mc/s. The total tuning capacitance being of the order of 12 $\mu\mu$F, the frequency-interval corresponding to 0.1 $\mu\mu$F detuning capacitance is about 1.25 Mc/s. Considering the flatness of the curve II of Fig. 4 corresponding to 0.1 $\mu\mu$F detuning capacitance, the proposed reduction of noise ratio obviously also applies to wide-band reception.

By optimal coupling and (if necessary) detuning of the input circuit the gain of the amplifier stage will become less than the optimal gain obtainable with this stage under favourable conditions with respect to gain. Though a high gain is not so important in itself, a low gain figure may cause an increase of overall noise ratio of the receiver as the noise introduced by the second stage could make an appreciable contribution to this ratio. Now, by a suitable feed-back from the output to the input of a stage its noise ratio is only altered to a minor and often negligible degree[6]. But stage gain may be considerably increased by such feed-back. Hence it may afford a suitable means of compensation for the loss of gain incurred by the coupling and detuning corresponding to a minimum noise ratio. Care should, of course, be taken that no appreciable additional contributions to the overall noise ratio are introduced by the feed-back circuit itself. Preferably, this should not incorporate unsuitable resistance elements.

In the above discussion only amplifier stages were considered. Similar means of noise-reduction may, however, be also applied to *mixer* stages[6], taking due account of noise correlation. If the first stage of a receiver contains a triode with the input signal as well as the local oscillator voltage acting between cathode and grid, reasoning similar to that of Sections 2 to 5 may be applied. The transadmittance S has to be replaced by the

conversion transadmittance S_c, this being in general complex at the present frequencies, as is S. The value of R_g, due to the action of the local oscillator voltage, will generally be *higher* than with amplifier operation. By the application of the coupling and detuning similar to those of Sections 3, 5 and 6 mixer stages of low noise ratio may be obtained, particularly suitable for the input of receivers at the present frequencies. A complete discussion of such mixer stages will be published elsewhere.

It should be pointed out that the application of A.V.C. by bias regulation might upset the results of the detuning and input-coupling under discussion. By the application of proper delay or of different methods of A.V.C., specially devised for cases like the present one, an increase of noise ratio as well as of selectivity due to A.V.C. may be avoided. It is hoped to deal with these points more fully in a future article.

APPENDIX

Noise Ratio and Noise Figures

It may be well to compare briefly the noise ratio, used in the present article, with some of the different definitions of noise figures, proposed in previous publications. The first of such figures [3, 4, 8] refers to the ratio of the mean-square noise voltage to the signal-voltage squared at the input electrodes of the first valve. The noise introduced by the valve itself, including its shot noise in the anode-circuit, was transferred to this input circuit by the well-known artifice of introducing a corresponding noise voltage generator of zero internal impedance in series with one of the valve's input terminals. The valve itself could then be assumed to be free of noise. The signal voltage is supplied by a signal generator of voltage V_a and of zero internal impedance in series with an ohmic resistance R_a. The available power of this signal source, i.e., the maximum power to be drawn from it for dissipation in a further circuit connected to its terminals, is $V_a^2/4R_a$. Now by the definition mentioned the available signal power, necessary to make the said ratio of mean-square noise voltage to signal-voltage squared at the valve's input terminals equal to unity, is used as a measure for the relative noise of the stage. Dividing this power by $kT\Delta f$ (T being the room temperature) the *noise-figure* N is obtained.

We shall show that our noise ratio (8) or (18) corresponds very closely to the said noise figure. In fact, the ratio of mean-square noise voltage to the signal voltage squared at the input terminals is *exactly* equal to the noise ratio (8) or (18). Hence, if this ratio is assumed to be unity, the expression between brackets of Equ. (18) coincides with the above noise figure N.

In a more recent definition [5, 13] the available noise power at the output terminals of a stage is divided by the available signal power at these terminals to obtain the noise ratio. The noise figure F of a stage is then said to be the ratio of its output noise ratio to the output noise ratio of the preceding stage. By the method of short-circuiting the output terminals our noise ratio coincides exactly with the ratio of available noise power to available signal power at the output. Hence, using the notation of Equ. (18) the noise figure F is given by:

$$F = \frac{kT\Delta f\left(\frac{T_a}{T} + \frac{T_g}{T}w\right)}{V_a^2/4R_a} \cdot \frac{V_a^2/4R_a}{kT_a\Delta f}$$

$$= 1 + \frac{T_g}{T_a}w = \frac{T}{T_a}N = N,$$

if $T_a = T$, as was assumed in the original definition of F.

Thus the quantity w which we have extensively used in the preceding sections is simply:

$$w = (F - 1)\frac{T_a}{T_g}$$

While the noise figure N corresponding to two subsequent stages of suffixes a and b, noise figures N_a and N_b and gain figures g_a and g_b is:

$$N = N_a + \frac{N_b - T_a/T}{g_a}$$

the corresponding relation in the F figures is:

$$F = F_a + \frac{F_b - 1}{g_a}$$

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TRANSIENT RESPONSE OF TUNED-CIRCUIT CASCADES*

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SUMMARY.—An analysis is given of the transient response of two, of four and then of any number N tuned circuits, all with the same resonance frequency, connected in cascade with a forward transmission coupling but no mutual coupling. It is shown that the envelope of the build-up or decay of a pulse applied at the resonance frequency is determined by the usual negative exponential term ($\epsilon^{-\beta t}$) applying to a single circuit, with a factor consisting of the first N terms of the series expansion of the corresponding positive exponential. The envelope of the transient component of the output signal when the applied frequency is not very close to resonance is given by $\epsilon^{-\beta t}$ multiplied by the Nth term of the same series expansion of $\epsilon^{\beta t}$. A comparison of the responses of tuned-circuit cascades with those of underived band-pass filters shows that for equal component qualities in the two cases, two tuned circuits in cascade are approximately equivalent, for pulse transmission applications, to a single-section band-pass filter using 50 per cent more components.

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   5.2 Dissipative filters.

References.

Appendix.

1. Introduction

THE use of a series of tuned circuits coupled together by mutual inductance, mutual capacitance, or resistance is well known, and the response of such an arrangement is analysed in various text-books and papers. The use of a cascade of tuned circuits which have transmission coupling but no mutual coupling is not so often referred to, however, although such an arrangement occurs frequently in practice. It is the purpose of the first parts of this paper to deduce the transient response of such a cascade, and it will be seen that very simple and elegant expressions are obtained. In spite of their simplicity, however, two independent searches of the literature have failed to reveal any previous publication of them.

It is also fairly well known that coupled tuned circuits are equivalent to certain types of conventional "3-element" band-pass filter[1]; although the latter are generally designed in accordance with Zobel's theory[2], it is nevertheless a fact that improved filtration can be obtained from single sections by a more empirical or *ad hoc* design such as is generally adopted for coupled tuned circuits.

The relationship between a cascade of tuned circuits without mutual coupling and a conventional band-pass filter is, however, of a different nature, and has probably not been investigated before. There is no direct circuit equivalence, but it is shown in the last part of this paper that there is a relationship in transient response, and that for the same Q-value of the resonant elements the overall performance of two tuned circuits in cascade is almost equivalent to that of an underived band-pass filter whole section, on a balance between transient response and steady-state discrimination. As the filter has three inductors and three capacitors, it is evident that the two tuned-circuit cascade effects an economy of components.

The same methods as are applied in this paper for determining the transient response of tuned-circuit cascades, and comparing them with band-pass filters, may be applied for determining the transient response of cascades of RC circuits and comparing them with low-pass filters. It is necessary only to replace the envelope functions by actual current or voltage functions when a d.c. signal is applied, and to regard a low-pass filter as equivalent to a band-pass with bandwidth equal to twice the low-pass cut-off

* MS. accepted by the Editor, February 1946.

frequency, and with midband frequency equal to zero.

Although the author's point of view is that of multi-channel pulse transmission systems in which the pulse is used to operate a relay* (e.g., a voice-frequency telegraph system or a multi-channel telephone trunk signalling system), an obvious application of the work is to multi-stage amplifiers with LC or RC interstage couplings. Such amplifiers are used in television with a.c. or d.c. pulses, but it must be pointed out that in the case of the LC circuits considered here, the Q-value is assumed high, so that the circuits are not suitable for d.c. pulses, being intended for a.c. pulses. Published treatments[10, 11] of cascaded stages in television video-frequency amplifiers (generally best treated by empirical or numerical methods) are listed in Section 7.

LIST OF SYMBOLS

R, L and C = the parallel components of a shunt-tuned circuit.

$\beta = \frac{1}{2RC} = \frac{\omega_0}{2Q}$ where ω_0 = resonance angular frequency, and $Q = \omega_0 RC = \frac{R}{\omega_0 L}$ for the parallel circuit used. β_1 and β_2 are also used where two tuned circuits of unequal Q are concerned.

ω = any applied frequency.

$x = \frac{\omega}{\omega_0}$

n = fractional bandwidth of filter $= \frac{\text{bandwidth}}{\text{mid-band frequency}}$.

$F_N(t)$ = output amplitude-time function of N tuned circuits in cascade.

N = number of tuned circuits in cascade.

2. Properties of a Single Tuned Circuit

A comprehensive account of the response, both steady-state and transient, of a single tuned circuit has been given by the author in another paper[4], although the subject is also dealt with in various text-books[5, 6]. Since the analysis of tuned-circuit cascades depends on the response of a single circuit, a summary is given here of the more important results. The tuned circuit considered is a parallel combination of R, L and C (as shown in Fig. 1) fed from a constant-current source, such as a pentode valve.

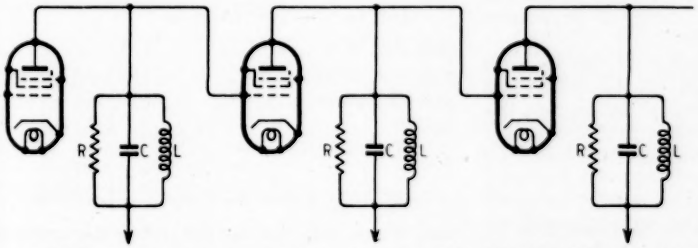


Fig. 1. *Cascade of tuned circuits without mutual coupling, and with blocking capacitors or coupling windings omitted for simplicity.*

We use $\beta = \frac{1}{2RC} = \frac{\omega_0}{2Q}$, and $Q = \frac{R}{\omega_0 L}$; we assume $Q \gg 1$; $\omega_0 = \frac{1}{\sqrt{LC}}$

ω = any applied angular frequency;
$x = \omega/\omega_0$
$\sin(\omega t + \phi)$ = suddenly applied signal,
ϕ = initial phase angle.

(a) Steady-state voltage attenuation ratio (relative to resonance frequency)

$$= Q(1/x - x) \qquad .. \quad .. \quad .. \quad (1)$$

provided x is sufficiently different from unity to give

$$Q^2 (1 - x^2)^2 \gg 1.$$

(b) Build-up of a pulse* applied at the resonance frequency

$$= (1 - \epsilon^{-\beta t}) \sin(\omega_0 t + \phi) \qquad .. \quad (2)$$

The corresponding decay curve is, of course, $\epsilon^{-\beta t} \sin(\omega_0 t + \phi)$.

The steady-state amplitude is assumed to be unity.

(c) The transient produced by a pulse applied at frequency ω, where ω is not very close to ω_0, depends on the initial phase angle, ϕ, of the applied signal at the moment of switching.

For an initial phase angle of zero, the transient has a peak value equal to that of the steady-state output, so that the combined output signal is (relative to unity peak at resonance)

$$\frac{\cos \omega t - \epsilon^{-\beta t} \cos \omega_0 t}{Q(1/x - x)} \qquad .. \quad .. \quad (3)$$

* Hence the emphasis placed on the "half-amplitude" values in later parts of the paper; for a fuller discussion see Reference 3.

* i.e., the response to a Heaviside Unit Step envelope.

It must be emphasized that the transient component has the frequency of resonance.

For an initial phase angle of $\pi/2$, the transient has a peak value equal to $1/x$ times that of the steady-state output, so that the combined output signal is

$$\frac{\sin \omega t - (1/x)\,\epsilon^{-\beta t} \sin \omega_0 t}{Q\,(1/x - x)} \quad .. \quad .. \quad (4)$$

As a rule the initial phase is uncontrolled, so that the maximum amplitude of the transient component is

(i) equal to the peak of the steady-state output for frequencies above resonance, and

(ii) equal to $1/x$ times the peak of the steady-state output for frequencies below resonance. Thus at very low frequencies, the maximum transient is relatively very large.

3. Two Tuned Circuits in Cascade

The cascade connection of tuned circuits is made so that the voltage across the first excites the next in such a manner that the total current through it is proportional to the voltage across the first circuit. This is conveniently achieved by driving each tuned circuit from a pentode valve, as shown in Fig. 1. An approximation to the condition would be if the tuned circuits were coupled by a high resistance. The time and frequency functions of the circuits are then independent and are given in Section 2. We are concerned only with the case of identical resonance frequencies.

The steady-state response of the cascade is obviously the product of the individual responses. If the first circuit has Q_1 and the second Q_2, then the variation of voltage attenuation ratio with frequency is given by

$$Q_1 Q_2\,(1/x - x)^2 \quad .. \quad .. \quad .. \quad (5)$$

relative to a ratio of unity at resonance.

The transient response of the cascade can be determined by means of the Superposition Theorem, which is most readily applied by means of Duhamel's Integral,[7]

$$F(t) = \int_0^t E(u)\,\frac{d}{d\,(t-u)}\,\phi\,(t-u)\,du \qquad (6)$$

where

$F(t)$ is the required output time function,

$E(t)$ is the voltage-time function impressed on the second circuit, i.e. the output of the first circuit.

$E(u)$ is the same function, but with the variable of integration u, replacing t.

$\phi(t)$ is the time function (or indicial admittance) of the second circuit.

Now if the functions (2), (3) or (4) are used in this integral, the process of integration is very laborious, and the results are extremely complicated, although perfectly rigid. It is proposed in this paper to simplify the process by adopting three ideas thus :—

(a) Treat the cases of applied frequency equal to and not equal to resonance frequency entirely separately, and ignore the case when they are almost equal, and

(b) Ignore the transient produced by the *steady-state* output from the first circuit being applied to the second circuit, in cases where the applied frequency is not equal to the resonance frequency. This is justified in most cases, as will be seen later. Thus the signal applied to the second circuit is always at the resonance frequency since the *transient* output from the first circuit is at this frequency. Then

(c) Since only the one (resonance) frequency is concerned now, we need only consider the envelope functions, which simplifies the integration enormously.

3.1 *Build-up of Resonance-frequency Pulse*

If the applied signal consists of a tone of the resonance frequency modulated by a Heaviside unit step (or a pulse with square envelope), we have, considering envelopes only

$$E(t) = 1 - \epsilon^{-\beta_1 t} \text{ from (2)}$$

and $\phi(t) = 1 - \epsilon^{-\beta_2 t}$

where β_1 corresponds to Q_1 of the first circuit and β_2 corresponds to Q_2 of the second circuit

$$\therefore \quad E(u) = 1 - \epsilon^{-\beta_1 u}$$

$$\frac{d}{d(t-u)}\,\phi\,(t-u) = \beta_2 \epsilon^{-\beta_2 (t-u)}$$

and if we call the output from the second tuned circuit $F_2(t)$

then

$$F_2(t) = \int_0^t (1 - \epsilon^{-\beta_1 u})\,\beta_2 \epsilon^{-\beta_2 (t-u)}\,du$$

$$= \int_0^t \beta_2 \epsilon^{-\beta_2 (t-u)}\,du - \int_0^t \beta_2 \epsilon^{-\beta_2 t - (\beta_1 - \beta_2) u}\,du$$

$$= 1 - \epsilon^{-\beta_2 t} + \frac{\beta_2}{\beta_1 - \beta_2}\,(\epsilon^{-\beta_1 t} - \epsilon^{-\beta_2 t}) \quad .. \quad .. \quad (7a)$$

$$= 1 + \frac{\beta_2}{\beta_1 - \beta_2}\,\epsilon^{-\beta_1 t} - \frac{\beta_1}{\beta_1 - \beta_2}\,\epsilon^{-\beta_2 t} \quad .. \quad .. \quad (7b)$$

It will be seen that this expression is indeterminate when $\beta_1 = \beta_2$, since the last term in (7a) is zero divided by zero. A different form is required for this special case, and can be obtained from (7a) by putting in the series expansion of the exponentials and then dividing out the common factor $(\beta_1 - \beta_2)$, or more simply by putting $\beta_1 - \beta_2 = 0$ in the line above (7a). This then gives the result

$$F_2(t) = 1 - \epsilon^{-\beta t}(1 + \beta t) \quad \text{when} \quad \beta_1 = \beta_2 \qquad (8)$$

This is very simple, and shows that the response of two identical tuned circuits is obtained simply by the modification of the exponential term by the factor $(1 + \beta t)$.

In the special case when one circuit has twice the Q-value of the other, we obtain from (7)

$$F_2(t) = (1 - \epsilon^{-\beta t})^2 \qquad (9)$$

where β is the value for the higher Q-value. This is interesting as being the square of the output function of the first circuit.

These build-up curves are illustrated in Fig. 2, where curve 1 shows the output of

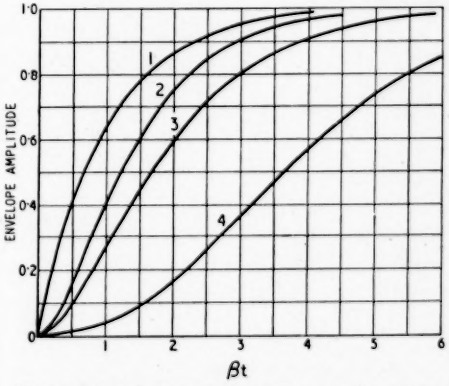


Fig. 2. *Build-up envelopes with applied pulse frequency equal to resonance frequency. Curve 1 = one tuned circuit; curve 2 = two tuned circuits, $Q_1 = 2Q_2$ or $2Q_1 = Q_2$; curve 3 = two tuned circuits, equal Qs; curve 4 = four tuned circuits, equal Qs.*

the first tuned circuit, curve 2 shows the output when $Q_1 = 2Q_2$, and curve 3 shows the output when $Q_1 = Q_2$. The time axis is taken in terms of βt to make the curves quite general. It will be seen that the effect of the second tuned circuit is to slow down the build-up, particularly at the beginning of the operation. In practical cases, it is generally the slope of the curve at the half-steady-state amplitude which forms the chief design consideration, and consequently the slopes of these three curves are shown in Table 1. It will be noticed that the addition of a second tuned circuit of equal Q-value decreases the slope by about 40 per cent and the time of build-up to 90 per cent of the steady-state amplitude is increased by about 70 per cent.

TABLE 1

Slope of Build-up Curve at Half-steady-state Amplitude

(Slope expressed as $\frac{dV}{dt}$, where $V = 1$ in the steady state)

Cascade	Slope
One tuned circuit	0.5β
Do. + another circuit of half the Q value	0.39β
Two tuned circuits of equal Q value	0.31β
Four tuned circuits of equal Q value	0.207β

An oscillogram of a 50 msec pulse transmitted through two tuned circuits with $Q \approx 18$ and resonance at 800 c/s is shown in the first oscillogram. This will be seen to confirm equation (8).

3.2 *Response to Pulse not at Resonance Frequency*

Here we will consider only the transient component of the output from the first tuned circuit. This, as was emphasized in Section 2, has the frequency of resonance. It will be seen below that it is not greatly attenuated by the second tuned circuit, whereas the steady-state (or more strictly the unit-step-modulated) component at the applied frequency is greatly attenuated. Thus any signal of the form of equations (3) and (4) produced by the latter component is of very small amplitude compared with the transient due to the original transient, and can generally be ignored. However, it is a simple enough matter to add it linearly if desired.

The analysis below is all in terms of unity peak amplitude of the transient at the output of the first circuit. Thus the function $E(t)$ is this time the envelope

$$E(t) = \epsilon^{-\beta_1 t}$$

so $E(u) = \epsilon^{-\beta_1 u}$

As before $\phi(t) = 1 - \epsilon^{-\beta_2 t}$

Thus the output $F_2(t) = \int_0^t \epsilon^{-\beta_1 u} \beta_2 \epsilon^{-\beta_2(t-u)} \, . \, du$

$$= \frac{\beta_2}{\beta_2 - \beta_1} (\epsilon^{-\beta_1 t} - \epsilon^{-\beta_2 t}) \quad .. \quad .. \quad (10)$$

For the case of equal Q-values, i.e. when $\beta_1 = \beta_2$, this is again indeterminate, and for this special case we obtain the solution

$$F_2(t) = \beta t \, \epsilon^{-\beta t} \quad .. \quad .. \quad .. \quad (11)$$

For the special case of one Q-value being twice the other, we obtain

$$F_2(t) = \epsilon^{-\beta_2 t} (1 - \epsilon^{-\beta_2 t}) \text{ when } \beta_1 = 2\beta_2$$

and $F_2(t) = 2\epsilon^{-\beta_1 t} (1 - \epsilon^{-\beta_1 t})$ when $\beta_1 = \beta_2/2$

$$.. \quad .. \quad (12)$$

These time functions are illustrated in Fig. 3, where curve 1 shows the output transient from the first circuit, curve 2 shows the output transient from two circuits when the first circuit has the higher Q and $\beta_1 = \beta_2/2$, and curve 3 shows the output from two circuits of equal Q. The peak values of the transients are 0.5 and 0.368 for the curves 2 and 3 respectively. The transient peak occurs at a time $2/3\beta_1$ seconds from switching in the case of unequal Q-values, and at a time $1/\beta$ seconds in the case of equal Q-values.

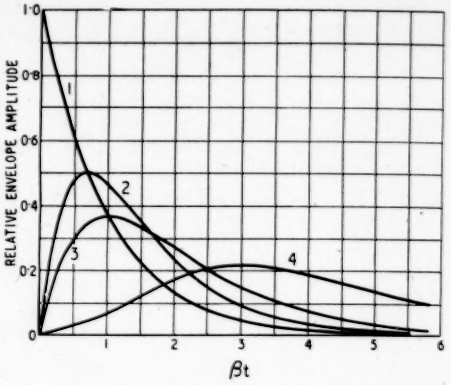
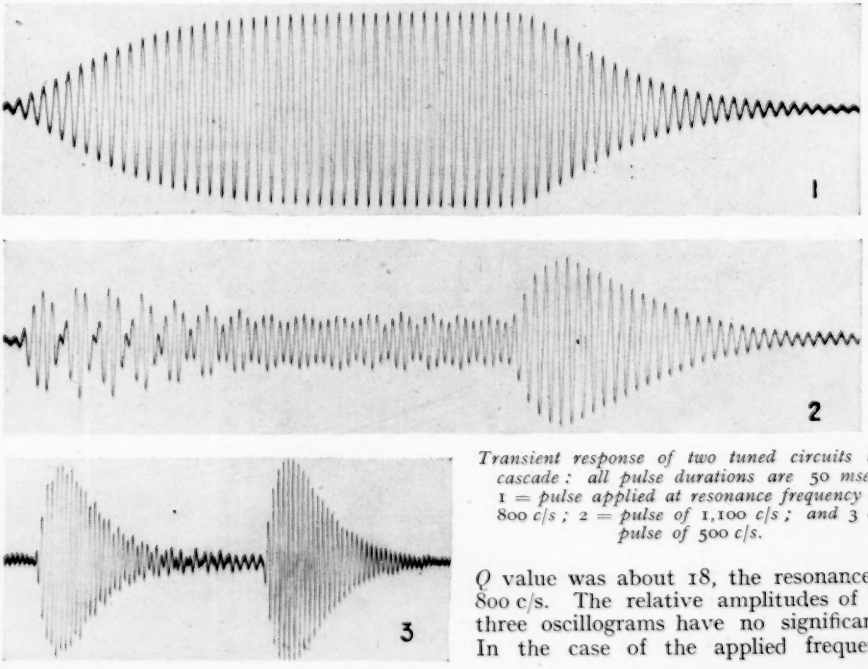


Fig. 3. *Transient envelopes for applied pulse frequency not close to resonance frequency. Curve 1 = one tuned circuit; curve 2 = two tuned circuits, $Q_1 = 2Q_2$; curve 3 = two tuned circuits, equal Qs; curve 4 = four tuned circuits, equal Qs.*

Oscillograms 2 and 3 show observed waveforms corresponding to the case of equal Q-values. As for Oscillogram 1 the Q value was about 18, the resonance at 800 c/s. The relative amplitudes of the three oscillograms have no significance. In the case of the applied frequency



Transient response of two tuned circuits in cascade: all pulse durations are 50 msec. 1 = pulse applied at resonance frequency of 800 c/s; 2 = pulse of 1,100 c/s; and 3 = pulse of 500 c/s.

= 1,100 c/s, the steady-state output is an appreciable proportion of the peak transient output. Apart from the beat produced on the make (due to the steady and transient components being of different frequency) there is the addition of a new transient to be considered, produced by the unit-step-modulated 1,100 c/s being applied to the second circuit. This transient is not in phase with the larger one, owing to the phase-shift at 1,100 c/s through the tuned circuit. If, as in the oscillograms, the initial phase is not controlled, there is a variation of peak transient amplitude due to the varying initial phase, and this is evident in Oscillogram 2. In Oscillogram 3 the steady-state and additional transients are practically negligible.

4. Any Number of Tuned Circuits in Cascade

Although, in practice, large numbers of identical tuned circuits are not likely to be connected in cascade, nevertheless the case is well worth examining because of the simplicity of the relationships. All Q-values are assumed equal.

Consider, to begin with, four circuits in cascade. We can regard this as two sets of two circuits, so that for Duhamel's Integral (Equ. 6) we have :—

$$E(t) = 1 - \epsilon^{-\beta t}(1 + \beta t)$$ for pulse at resonance frequency

and $E(t) = \beta t\, \epsilon^{-\beta t}$ for pulse not at resonance frequency

$$\phi(t) = 1 - \epsilon^{-\beta t}(1 + \beta t)$$

so that

$$\frac{d}{d(t-u)}\phi(t-u) = \beta^2 t\, \epsilon^{-\beta t + \beta u} - \beta^2 u\, \epsilon^{-\beta t + \beta u}$$

Calling the output time function $F_4(t)$, we have, for the first case,

$$F_4(t) = \int_0^t [1 - \epsilon^{-\beta u}(1 + \beta u)] \, [\beta^2 t\, \epsilon^{-\beta t}\, \epsilon^{\beta u} - \beta^2 u\, \epsilon^{-\beta t}\, \epsilon^{\beta u}]\, du$$

which, on integrating term by term, gives

$$F_4(t) = 1 - \epsilon^{-\beta t}\left(1 + \beta t + \frac{\beta^2 t^2}{2} + \frac{\beta^3 t^3}{6}\right) \quad .. \quad .. \quad (13)$$

as the output from four circuits when the applied pulse is at the resonance frequency.

For the second case,

$$F_4(t) = \int_0^t \beta u\, \epsilon^{-\beta u}(\beta^2 t\, \epsilon^{-\beta t}\, \epsilon^{\beta u} - \beta^2 u \epsilon^{-\beta t}\, \epsilon^{\beta u})\, du = \frac{\beta^3 t^3}{6}\, \epsilon^{-\beta t} \quad .. \quad .. \quad (14)$$

giving the output from four circuits when the applied pulse is not at the resonance frequency. For a case such as this, of four circuits, the effect of the steady-state component can fairly safely be ignored.

Graphs of the functions (13) and (14) are shown as curve 4 on Figs. 2 and 3 respectively, where they can be compared with the results for two circuits in cascade. The slope of the curve at half-steady-state amplitude for the pulse at resonance frequency is compared in Table 1 with the earlier results, and it will be seen that the addition of two circuits to two circuits decreases the slope by 33 per cent. The peak amplitude of the transient when the applied frequency is not the resonance frequency is 0.225 of the peak transient amplitude out of the first circuit.

Consider now a number N of circuits in cascade. It can be shown by performing a few integrations for numbers of circuits other than 2 or 4 that the general results are of the form :—

For applied pulse at resonance frequency*

$$F_N(t) = 1 - \epsilon^{-\beta t}\left(1 + \beta t + \frac{\beta^2 t^2}{2!} + \frac{\beta^3 t^3}{3!} + \ldots + \frac{\beta^{N-1} t^{N-1}}{(N-1)!}\right) \quad .. \quad (15)$$

and for applied pulse not at resonance frequency

$$F_N(t) = \frac{\beta^{N-1} t^{N-1}}{(N-1)!}\, \epsilon^{-\beta t} \quad .. \quad .. \quad (16)$$

These results are rather striking, and yield the following conclusions :—

(a) When there is an infinite number of circuits, the last term of equation (15) is the expansion of $\epsilon^{\beta t}$, so that $F_\infty(t) = 0$, and build-up never takes place. This is obviously true from physical considertations.

(b) For any given value of N, the maxi-

---

* This expression for $F_N(t)$ is actually the expansion of the Incomplete Gamma Function

$$\frac{1}{(N-1)!}\int_0^{\beta t} \epsilon^{-\beta t}\, (\beta t)^{N-1}\, d(\beta t)$$

so that numerical cases can be worked out by reference to tables of this function.[13] This result has been published (since the present paper was written) by Eaglesfield[12 14], whose method of derivation is completely different from that given here.

The expression (15) is also the same as Poisson's Exponential Probability Summation, and numerical cases can be determined very quickly by reference to tables published by Molina[15], or to charts, published originally by Campbell[16] and Thorndike,[17] and republished by Doust and Josephs.[18]

mum value of equation (16) is found to occur when

$$t = \frac{N-1}{\beta} \qquad .. \quad .. \quad .. \quad (17)$$

which gives an apparent delay time to the peaks of $1/\beta$ seconds per tuned circuit.

(c) The maximum amplitude of equation (16) [occurring at the time given by (17)] is

$$\frac{(N-1)^{N-1}}{(N-1)!}\epsilon^{-(N-1)} \qquad .. \quad .. \quad (18)$$

Fig. 4 shows a graph of the relationship between this maximum value of the transient and N. It will be seen that the transient

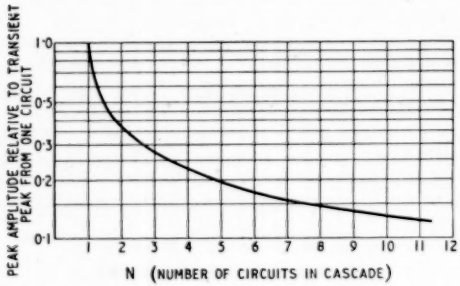

PEAK AMPLITUDE RELATIVE TO TRANSIENT PEAK FROM ONE CIRCUIT
1·0
0·5
0·3
0·2
0·1
1 2 3 4 5 6 7 8 9 10 11 12
N (NUMBER OF CIRCUITS IN CASCADE)

Fig. 4. *Amplitude of transient peak in relation to number of tuned circuits in cascade.*

attenuation of the cascade is increased very little by adding circuits after the fourth or fifth. An increase from 2 to 4 increases the transient attenuation by only 4.8 db.

5. Comparison of Tuned-Circuit Cascades with Band-Pass Filters

As stated in Section 1, there is no direct circuit equivalence between tuned-circuit cascades and band-pass filters, but a comparison of their performances can be made, since to a large extent the frequency function

$$(1/x - x)$$

determined the performance of both.

In practical systems it is likely that the determining design feature will be the slope of the build-up curve at half-steady-state amplitude for applied pulses of the resonance or midband frequency. Thus the comparison is made strictly on this basis; i.e., that the slope must be the same for any circuit combination considered.

There is no need to set out in detail how all the comparison values for tuned-circuit arrangements are derived; all the necessary data has already been given in preceding sections. In the case of band-pass filters, only underived sections are considered, and the derivations of the expressions used below are largely given in other papers by the present author[8, 9]. A more detailed account of the derivations is given in the Appendix.

5.1 *Non-dissipative Filters*

The comparison is made in Table 2, which is self-explanatory. The bandwidth of the filter is $n\omega_0$ radians/sec, where ω_0 is the midband angular frequency. It is assumed that the slope of the midband build-up curve of the filters is $\frac{n\omega_0}{2\pi}$ for one or two sections, which is true to an accuracy of about 15 per cent. [See reference (9), Section 2.3]. If then this is equated to the tuned-circuit slopes given in Table 1, a

TABLE 2

Comparison of Underived Band-pass Filter Performance with Cascaded Tuned Circuit Response

(Comparison based on all arrangements having the same slope of build-up curve at half-steady-state amplitude when applied frequency is equal to midband or resonance frequency)

	1-section filter	2-section filter	1 tuned circuit	2 tuned circuits (equal Q values)	4 tuned circuits (equal Q values)
Transient peak Discrimination (Voltage Ratio)	$(\pi/2n)(x - 1/x)$	$(\pi/n)(x - 1/x)$	$(\pi/2n)(x - 1/x)$	$\frac{0.85\pi}{n}(x - 1/x)$	$\frac{0.92\pi}{n}(x - 1/x)$
Steady-state Discrimination (Voltage Ratio)	$(1/n^3)(x - 1/x)^3$	$(1/n^6)(x - 1/x)^6$ very approximately	$(\pi/2n)(x - 1/x)$	$\frac{0.95}{n^2}(x - 1/x)^2$	$\frac{0.18}{n^4}(x - 1/x)^4$
Time required to attain 90 per cent steady-state Amplitude	approx. $\frac{2.2\pi}{n\omega_0}$	$\frac{2.9\pi}{n\omega_0}$	$\frac{2.3\pi}{n\omega_0}$	$\frac{2.45\pi}{n\omega_0}$	$\frac{2.8\pi}{n\omega_0}$

relationship between Q (for the tuned circuits) and n is established, thus

$$Q = \frac{2\pi A}{n} \quad .. \quad .. \quad .. \quad .. \quad (19)$$

where $A = 0.25$ for one tuned circuit,
$= 0.155$ for two tuned circuits,
$= 0.103$ for four tuned circuits.

This value of Q is then substituted in the various tuned-circuit equations.

(Note $\beta = \omega_0/2Q$).

The features compared are those which affect the design of multi-channel pulse transmission systems. The actual shape of the various envelopes of the pulses or transients are not compared, although there is a certain amount of correspondence here too.

From Table 2 it will be seen that there is no real general identity between any tuned circuit cascade and a filter. If transient attenuation (e.g., interference suppression or separation from adjacent channels) alone is important, then a single tuned circuit is as good as a 1-section filter using three times as many components. But if steady-state attenuation has to be considered (as it generally must be), then roughly three tuned circuits are required to be equivalent to a 1-section filter.

5.2 *Dissipative Filters*

The effect of dissipation in the filter components has been ignored in Table 2. If the filters have a narrow bandwidth, dissipation will have a large effect in practical cases where ordinary inductors are used. If we take the case in which the filter inductors have the same Q-value as the tuned-circuit inductors, then comparing two tuned circuits with a one-section filter, we have, from equation (19), $Q = \frac{0.31\pi}{n} \approx 1/n$. It was shown in an earlier paper[8] that the steady-state midband loss ratio of a one-section underived filter is approximately

$$1 + 2/Qn$$

which in the present case is approximately $1 + 2 = 3$, i.e. the steady-state midband loss is about 10 db, irrespective of the actual bandwidth or Q value. This quantity must therefore be subtracted from the steady-state discrimination shown in Table 2. For the transient condition this does not apply, because the loss is added to the signal whatever the applied frequency may be, since the transient is of midband frequency in any case.

It will be seen, then, that if the quality of the components is the same for both tuned-circuit cascade and filter, then a two-tuned-circuit cascade will approximate to the same steady-state discrimination as a one-section filter over at least that part of the band not too remote from the pass-band (which is the most critical part in most practical instances), and will have a superior transient discrimination over the whole band. It can therefore be concluded that two tuned circuits in cascade are, on the balance, roughly equivalent in overall performance to a one-section filter using 3 inductors and 3 capacitors; there is a consequent economy of components in using the cascade connection.

APPENDIX 1

Derivation of the Expressions in Table 2.

The expressions for transient and steady-state attenuation of the tuned circuit cascades of 1, 2 and 4 circuits are readily derived by the insertion of equation (19) into the various equations given earlier in the paper, e.g. equations (1) and (5) for the steady-state, and equations (3), (11) and (14), or (18), (or alternatively Fig. 3) for the transient state. The time required to attain 90 per cent steady-state amplitude is taken from Fig. 2.

For the band-pass filters, the steady-state attenuation is obtained from Reference 8, Part 1, Section 2, equation (4). This gives the insertion loss of one whole section between terminating resistances equal to the design resistance. Therefore, in taking the square of this as the loss ratio for two sections, an approximation is made because the effect of the impedance match at the junction between sections is not allowed for.

The transient peak attenuation (or discrimination) is obtained from Reference 9; Section 2.2, equation (5), gives the result shown to apply to a two-section filter; and from Section 2.3 and Fig. 1 of the same reference we get the fact that the attenuation is roughly half as great for a single-section filter.

It is to be noted that in all these comparisons the effect of initial phase angle is neglected; it is assumed the angle is zero, corresponding to an applied signal $\sin \omega t$. It was shown in Section 2 of the present paper that the transient peak for tuned circuits is $1/x$ times as large when the initial phase angle is $\pi/2$. It is shown in Section 2.2, equation (8) of Reference 9 that the same relationship applies in the case of band-pass filters. Thus the comparison is unchanged whatever initial phase angle is taken.

The time required for filter build-up curves to reach 90 per cent steady-state amplitude is taken from the results given in Reference 9.

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[2] O. J. Zobel, "Theory and Design of Uniform and Composite Electric Wave Filters," *Bell Syst. Tech. J.*, Vol. 2, p. 1 (1923) or standard text-books on Transmission Networks, e.g. ref. 5.

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[5] E. A. Guillemin, "Communication Networks," Vol. 1, p. 92 onwards.

[6] Massachusetts Institute of Technology, "Electric Circuits," p. 356 onwards (John Wiley).

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[13] K. Pearson, "Tables of the Incomplete Gamma Function."

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[17] F. Thorndike, *Bell Syst. Tech. J.*, Vol. 5, p. 604, Oct. 1926, also Bell Monograph B220.

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CORRESPONDENCE

Letters of technical interest are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Carrier-Frequency Amplifiers

To the Editor, "Wireless Engineer."

SIR,—In a recent paper ("*Wireless Engineer*," Vol. 23, No. 271, pp. 96–102, April, 1946) I discussed the transient response of a carrier-frequency amplifier to a sudden change in the frequency of the carrier. Professor van der Pol has drawn my attention in private correspondence to a difficulty in the analysis. The point is as follows:

If an admittance $A(p)$ is subjected to a voltage which for time t negative is sinusoidal in form with angular frequency ω_0, and for t positive has an angular frequency ω_0', then the current in the admittance is given by

$$i = (1-P)\ \cos \omega_0 t + Q \sin \omega_0 t + P_1 \cos \omega_0' t - Q_1 \sin \omega_0' t$$

This is equation (5) of the original paper; the coefficients P, Q, P_1, Q_1 are functions of t and are given by

$$P(t) + jQ(t) = \frac{A(p + j\omega_0)}{A(j\omega_0)}\,\mathbf{1}$$

$$P_1(t) + j\,Q_1(t) = \frac{A(p + j\omega_0')}{A(j\omega_0)}\,\mathbf{1}$$

In these expressions, p is the differential operator d/dt, $\mathbf{1}$ is the Heaviside step function, and $j = \sqrt{-1}$.

To obtain the effective frequency of i, the following definition is used:—

$$\text{Effective Frequency} = \frac{\text{Average of } di/dt}{\text{Average of } i}$$

the averages being root-mean-square.

In the original paper the assumption was made that for taking the averages a duration of time could be chosen, which was long compared to $1/\omega_0$ and $1/\omega_0'$, but during which P, etc., were sensibly constant. But in calculating the averages it was also implicity assumed that the duration of time was long compared to $1/(\omega_0 - \omega_0')$. A little consideration will show that, in any practical application of the equations to a network, this last assumption is likely to conflict with the assumption that P, etc., remain constant during the time interval. For if the network has a fairly well-defined pass-band, and ω_0 and ω_0' are placed as far apart as possible (on the edges of the band), this will make $1/(\omega_0 - \omega_0')$ as small as possible.

But it is known, in a general way, that the transition time is of the order of the reciprocal of the pass-band, so that P, etc., will not be constant during $1/(\omega_0 - \omega_0')$; even less will they be constant during a time interval which is long compared to $1/(\omega_0 - \omega_0')$.

In order to simplify the expressions in the averages, it is now assumed that the time interval for taking the averages is short compared to $1/(\omega_0 - \omega_0')$. In squaring the expression for i it is now necessary to retain the cross-product terms which were previously neglected. They lead to terms in $\cos(\omega_0 - \omega_0')t$ and $\sin(\omega_0 - \omega_0')t$, and with the new assumption they are approximately unity and zero respectively.

Recalculating the averages on this basis, it will be found that

$$\begin{aligned} 2\,\overline{i^2} &= (1-P)^2 + Q^2 + P_1^2 + Q_1^2 \\ &\quad + 2\,(1-P)\,P_1 - 2\,QQ_1 \\ &= 1 + 2\,(P_1 - P) + (P_1 - P)^2 + (Q_1 - Q)^2. \end{aligned}$$

Now since $(\omega_0 - \omega_0')$ is being taken very small, it is clear that $(P_1 - P)$ and $(Q_1 - Q)$ are also very small.

$$\text{Thus } 2\,\overline{i^2} = 1$$

That is to say, the amplitude remains constant during the transition.

Handling $\overline{\frac{di}{dt}^2}$ in the same way, we get for the effective frequency ω:—

$$\begin{aligned} \omega^2 &= \left[(1-P)^2 + Q^2\right]\omega_0^2 + \left[P_1^2 + Q_1^2\right]\omega_0'^2 \\ &\quad + 2\,(1-P)\,P_1\,\omega_0\,\omega_0' - 2\,QQ_1\,\omega_0\,\omega_0' \end{aligned}$$

By assuming $(\omega_0 - \omega_0')$, $(P_1 - P)$, $(Q_1 - Q)$ all small, this is reduced to the simple expression:

$$\omega = \omega_0 + (\omega_0' - \omega_0)\,P.$$

That is, the transient modulation ratio is P, a quantity which can be seen from its definition to vary from zero at $t = 0$ to unity at $t = \infty$.

It should be noted that the new assumption, that ω_0' is nearly equal to ω_0, is equivalent to postulating a small depth of frequency modulation.

Now it was shown in the original paper (Equation 14*b*) that the transient modulation ratio for a small amplitude modulation was also P.

We thus have the interesting result that the transient modulation ratio is the same for either

frequency or amplitude modulation, the modulation being small in each case, and in the form of a unit step. But it is possible to go further: since the transient modulation ratio is an impedance it follows that the output modulation for any waveform of input modulation can be obtained from the step response (Borel's Theorem). It thus follows that the modulation ratio is the same for both amplitude and frequency modulation, whatever the waveform of the input modulation, provided it is small.

It may be well to repeat this statement in a slightly different way.

If a carrier frequency, with a small amplitude modulation, is applied to the input terminals of a linear four-terminal network, then the amplitude modulation at the output terminals will differ from the input modulation. Also, if the carrier has a small frequency modulation, the output frequency modulation will differ from the input modulation. The distortion of the modulation is independent of the type of modulation, amplitude or frequency, but depends entirely on the network.

The actual waveform of the output modulation can be calculated, if the network and the waveform of the input modulation be known. But unless at least one of these is simple in form, the calculation would be laborious.

In the original paper, equations (6), (7), (8), and Figs. 4, 5, should be disregarded. They have no practical application.

However, the curves given previously for amplitude modulation apply directly to frequency modulation. They will be found in *Wireless Engineer*, Vol. 22, No. 266, p. 523, November 1945, and also Vol. 23, No. 270, p. 67, March 1946.

I am extremely grateful to Professor van der Pol for drawing my attention to this matter, and to Dr. F. L. Stumpers for communicating his notes on my paper, in which by a slightly different treatment he reached substantially the result given here.

Bournemouth. C. C. EAGLESFIELD.

BOOK REVIEWS

Introduzione alla Radiotelemetria (Radar)

By Prof. Ugo Tiberio. Pp. 277 and 137 Figs. Editore Rivista Marittima Roma. 300 Lira.

The author is Professor of Electrotechnics at the Royal Naval Academy and has evidently made a special study of radar. The book presupposes a general knowledge of radio principles and practice and begins straight away with references to the experiments of Breit and Tuve and of Appleton and Barnett. In the third line there is a reference to Heavisyde (*sic*) and the *imosfera* (presumably a misprint for the ionosfera), which is a bad start. The book is divided into three parts and at the end a footnote states that there is a fourth part which will be published separately. Part I is a general introduction to the methods of radiolocation, the use of cathode-ray tubes, the reflection of waves, etc. Part II is concerned with the circuit arrangements for generating the impulses or waves, special short-wave aerials and reflectors, and the corresponding receiving apparatus. Part III is devoted to the theory of the transmission and reflection, polar curves of radiation, pulse distortion, and allied problems.

G. W. O. H.

Télétransmissions par Ondes Porteuses dans les Réseaux de Transport d'Energie a Haute Tension

By ANDRÉ CHEVALLIER. Pp. 111 + x and 124 Figs. Dunod, 92, rue Bonaparte, Paris (VI).

The development of an underground 60 000-volt network in 1920 necessitated new methods of protection which were developed by M. Fallou. This work, interrupted by his death, has since been taken up and developed successfully. The book is based on courses of lectures given by the author since 1942 at the Ecole Supérieure d'Electricité, it deals with the superposition of high-frequency currents on the high-voltage grid, their modulation, transmission, etc. These high-frequency carrier waves are used to transmit the necessary information, measurements, controls, etc. After explaining modulation and detection, the author considers the various ways in which a 3-phase line can be used to transmit the high-frequency carrier wave, and develops the transmission formulae for the various arrangements. The capacitors employed to couple the h.f. apparatus to the high voltage line, which may be at 220 kV, present some problems; high-pass and band-pass filters are considered as are also rejector circuits, spark-gap protective devices and quartz filters. A chapter is devoted to the problems involved in using such a system for telephony and another chapter to its application to various types of protection. The book contains a large number of diagrams of connections, graphs, etc.; it should certainly be studied by those interested in the protection of large high-voltage networks by means of superposed high-frequency carriers.

G. W. O. H.

University of Glasgow

THE retirement of Professor G. W. O. Howe from the James Watt Chair of Electrical Engineering is taking place at the end of the present session. We are asked to say that a Presentation to mark the occasion is being arranged and contributions are invited from old students and other friends.

The arrangements are in the hands of Dr. A. J. Small, Electrical Engineering Dept., The University, Glasgow, W.2, to whom contributions should be sent. H.S.P.

Parsons Memorial Lecture

THE Parsons Memorial Lecture for 1946, arranged by the Institute of Civil Engineers, will be given on September 26th, 1946, at 5.30 p.m. at the Institution of Civil Engineers, Gt. George St., London, S.W.1. The lecture is by Sir Hugh Chance and is on "Recent Developments in Optical Glass Manufacture".

Members of the Institution of Electrical Engineers are invited to attend.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

ACOUSTICS AND AUDIO-FREQUENCY CIRCUITS AND APPARATUS

575 306.—Visual warning signal to prevent any improper manipulation of the controls of an automatic or record-changing gramophone, during the playing period.

Industriakt Luxor. Convention date (Sweden) 20th February, 1943.

575 327.—Valve potentiometer for stabilizing the voltage across a variable resistance-load, and particularly for controlling the volume of an electronic musical organ.

A. H. Midgley. Application date 15th December, 1943.

575 443.—Balanced bridge circuit, including temperature-compensating coils, for detecting landmines, booby-traps, or the presence of submarines at sea.

Cinema-Televison Ltd. and S. S. West. Application date 29th March, 1943.

575 574.—Coupling arrangement in which fixed primary and secondary coils are associated with a fixed intermediate coil and with a movable non-magnetic cylinder, in order to simulate a resistance coupling, say for detecting buried metals.

Hazeltine Corporation (assignees of L. F. Curtiss). Convention date (U.S.A.) 31st December, 1942.

575 671.—Apparatus for observing by oscilloscope the nature of acoustic and other transients and for making magnetic records for reproduction.

The Brush Development Co. (assignees of S. J. Begun). Convention date (U.S.A.) 1st January, 1942.

AERIALS AND AERIAL SYSTEMS

575 097.—Moulded terminal block of polythene for coupling and supporting say a coaxial feed-line to dipole aerials.

A. D. Ferguson and Metropolitan-Vickers Electrical Co. Ltd. Application date 26th November, 1941.

575 534.—Aerial for radiating micro-waves consisting of an unsheathed rod of dielectric, part of its length being of uniform and the rest of tapering cross-section.

Western Electric Co. Inc. Convention date (U.S.A.) 17th December, 1942.

DIRECTIONAL WIRELESS

575 154.—Blind-landing system in which the transmitter radiates a frequency-modulated wave and the aeroplane develops a control current based on the difference between the direct wave and that received after reflection from the ground.

Hazeltine Corp. (assignees of H. M. Lewis). Convention date (U.S.A.) 26th February, 1942.

575 430.—Highly-directive aerial array, including phase-inverting or suppressor elements, particularly suitable for relaying frequency-modulated programmes from the studio to the main transmitter.

The British Thomson-Houston Co. Ltd. Convention date (U.S.A.) 31st December, 1942.

575 432.—Radiolocation equipment in which the common transmitting and receiving aerial is formed by the flared end of a wave-guide, the body of which comprises automatic blocking and acceptor circuits.

Western Electric Co. Inc. Convention date (U.S.A.) 30th January, 1943.

RECEIVING CIRCUITS AND APPARATUS

(*See also under Television*)

575 188.—Receiver in which the signal and " noise " voltages are developed across impedances having long and short time-constants, respectively, in order to balance out the disturbances.

The British Thomson-Houston Co. Ltd. Convention date (U.S.A.) 31st October, 1942.

575 250.—Time-base voltage-generator, say for a c.r. tube, in which a valve is negatively back-coupled through a time-constant network which may be of the integrating or differentiating type.

A. C. Cossor Ltd. and J. W. Whiteley. Application date 17th February, 1942.

575 329.—Push-button tuning system for a superhet set in which each selected frequency depends upon two tuned circuits, one pre-set and the other continuously variable.

A. C. Cossor Ltd. and D. A. Bell. Application date 21st December, 1943.

575 493.—Portable receiver fitted with tuning controls which are so arranged that they can be conveniently operated when the set is being carried by one hand.

Philco Radio and Television Corp. (assignees of R. J. Whipple). Convention date (U.S.A.) 11th February, 1943.

575 583.—Visual tuning indicator, requiring no d.c. amplifier, for frequency or phase-modulated signals.

Marconi's W.T. Co. Ltd. (assignees of M. G. Crosby). Convention date (U.S.A.) 27th March, 1943.

575 627.—Construction of flat, ribbed valve socket or holder, particularly for the midget type of tube.

Cinch Manufacturing Corp. (assignees of S. M. Del Camp). Convention date (U.S.A.) 16th December, 1942.

575 639.—Amplifier consisting of single valve-stages coupled in cascade through anode and cathode impedances which are selected to produce zero phase-displacement over a range of frequencies.

B. M. Hadfield. Application date 14th December, 1943.

575 796.—Making a permanent record of short time-intervals, as in radiolocation, by means of a device in which a stylus is traversed bodily over a time-base resistance or potential-divider.

N. P. Hinton and C. S. Wright. Application date (for a secret patent) 14th May, 1942. Published 30th November, 1945.

TELEVISION CIRCUITS AND APPARATUS

For Transmission and Reception

575 060.—Television C.R. tube in which the transparency of a screen, subjected to external light, is modified by the heating-effect of the scanning-stream.

G. Liebmann and Cathodeon, Ltd. Application date 20th November, 1941.

TRANSMITTING CIRCUITS AND APPARATUS

(See also under Television)

575 023.—Construction of wave-guide designed to admit of bending or twisting without loss of transmission-efficiency.

Callenders Cable and Construction Co. Ltd., D. T. Hollingsworth and C. W. J. Morley. Application date 6th July, 1944.

575 156. — Frequency-discrimination circuit for stabilizing a short-wave oscillator say between modulation intervals.

The General Electric Co. Ltd. and J. B. Lovell-Foot. Application date 18th June, 1943.

575 228.—Construction of wave-guide the transmission-efficiency of which is unimpaired by flexing or by mechanical vibration.

W. D. Allen. Application date 8th March, 1944.

575 347.—Screwdriver device for adjusting the tuning of radio components which are normally enclosed or sealed against the high humidity of tropical climates.

Standard Telephones and Cables Ltd. and G. Newton. Application date 1st March, 1944.

575 395.—The use of a thermister—a device having a temperature coefficient of resistance—for stabilizing the frequency of a two-stage resistance-coupled oscillator.

Standard Telephones and Cables Ltd., G. C. Hartley and J. A. B. King. Application date 15th March, 1944.

575 511.—Purely-resistive terminating-device for a coaxial transmission-line, including a crystal for measuring the shunt voltage.

Marconi Instruments Ltd. and E. W. Hunt. Application date 1st February, 1944.

575 577.—Wave-guide comprising an adjustable phase-shift section for transmitting a wide band of frequencies, and rotary joints for converting the waves from linear to circular polarization.

Western Electric Co. Inc. Convention date (U.S.A.) 3rd November, 1942.

575 739.—Variable-impedance device for a wave-guide in which a diaphragm acting as a reactance is associated with one or more adjustable probes acting as capacitances.

The British Thomson-Houston Co. Ltd., L. W. Brown and J. H. Nicoll. Application date 30th March, 1944.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

575 213.—Speech-free signalling and supervisory circuits arranged between the frequency zones in a carrier-wave multi-channel system.

Automatic Telephone and Electric Co. Ltd. and L. J. Murray. Application date 7th March, 1944.

575 384.—Compact network, simulating a double transmission-line, for the generation of periodic square-shaped signalling impulses.

The British Thomson-Houston Co. Ltd. and K. J. R. Wilkinson. Application date 3rd February, 1942.

575 462.—Two-way signalling system in which the same aerial serves to transmit morse and to receive incoming speech or other signal in the intervals of transmission.

Standard Telephones and Cables Ltd. and E. G. Seath. Application date 17th March, 1944.

CONSTRUCTION OF ELECTRONIC DISCHARGE DEVICES

574 758.—Arrangement of the cooling-fins and transverse baffle-plates for a high-powered electron-discharge tube.

Standard Telephones and Cables, Ltd. and W. T. Gibson. Application date 15th February, 1944.

574 934.—Toroidal resonator, of the rhumbatron type, wherein bimetallic tuning control is operated (a) by the rise in working temperature, or (b) by the deliberate application of heat.

The M-O Valve Co., Ltd., N. L. Harris and J. W. Ryde. Application date 22nd April, 1940.

574 878.—Electron-discharge tube of the "light-transformer" type, designed to facilitate the application of a photo-sensitive coating to the cathode in the course of manufacture.

J. D. McGee. Application date 17th November, 1942.

574 935.—Electron-discharge tube designed to facilitate the connection of an external concentric feed-line to one of the electrodes, particularly for velocity-modulation.

The M-O Valve Co., Ltd., N. L. Harris and R. W. Sloane. Application date 3rd June, 1940.

574 943.—Coupling circuit for taking power from magnetron oscillators of the kind in which the anode-block comprises a number of cavity resonators.

Western Electric Co., Inc. Convention date (U.S.A.) 1st May, 1942.

574 967.—Electron-discharge tube in which the electron stream is first deflected to and fro through a perforated screen to produce a frequency-multiplying effect before being fed to a hollow resonator or rhumbatron.

The M-O Valve Co., Ltd., J. W. Ryde and R. W. Sloane. Application date 8th October, 1940.

574 968.—Tuning a velocity-modulating device, or rhumbatron, lying partly within an electron-discharge tube, by means external to the evacuated tube.

The M-O Valve Co., Ltd., N. L. Harris and R. F. Proctor. Application date 8th October, 1940.

574 969.—Construction and arrangement of the hollow-resonator in an electron-discharge tube designed for velocity-modulation.

The M-O Valve Co., Ltd. and N. L. Harris. Application date 8th October, 1940.

574 972.—Short-wave oscillator wherein the electron stream is reflected back through the original bunching resonator across a path of specified field-gradient.

S. Hill. Application date 17th January, 1941.

574 997.—Construction of the target or reflecting electrode associated with a rhumbatron resonator in order to facilitate degassing of the electron-discharge tube.

J. E. I. Cairns. Application date 8th August, 1941.

575 123.—Discharge-tube in which the grids of two coaxial resonators are relatively adjusted by magnetic or other means external to the tube.

Standard Telephones and Cables, Ltd. (assignees of C. V. Litton). Convention date (U.S.A.) 20th April, 1940.

575 249.—Electron-multiplier with means for focusing the primary and secondary electrons in order to increase the ratio of transconductance to plate current.

Marconi's W.T. Co. Ltd. (communicated by Radio Corporation of America). Application date 10th September, 1941.

575 320.—Process for making "honeycomb" electrodes or grids intended to respond to the action of a high-speed electron-beam.

The British Thomson-Houston Co. Ltd. and W. J. Scott. Application date 3rd November, 1943.

575 747.—Process for making the flattened viewing-end of the glass bulb of a cathode-ray tube.

Chance Brothers Ltd., W. M. Hampton and N. J. B. Raymond. Application dates 4th April and 10th August, 1944.

575 824.—Moulding process for making metal-to-glass seals, say in the manufacture of electron-discharge devices.

Standard Telephones and Cables Ltd. and T. W. Wingent. Application date 13th April, 1944.

575 962.—Cathode-ray tube in which the end of the glass bulb is lenticular with one of its surfaces cylindrical, in order to minimize distortion.

Chance Bros. Ltd., J. G. Holmes and J. English. Application date 17th March, 1944.

SUBSIDIARY APPARATUS AND MATERIALS

574 840.—Equipment, including mechanical gear, for selecting and stabilizing the generation of a number of "slave" frequencies derived from a "master" oscillator, such as a piezo-electric crystal.

The General Electric Co., Ltd. and A. S. Gladwin. Application date 1st March, 1943.

574 916.—High-frequency induction-heater designed to offset the magnetomotive forces set up within the work.

Standard Telephones and Cables, Ltd. (assignees of V. W. Sherman). Convention date (U.S.A.) 25th February, 1943.

574 996.—The use of calcium-fluoride powder as a stabilizing agent for the spark-gap of a high-frequency ignition system.

The Plessy Co. Ltd. and H. V. G. Stubbs. Application date 1st July, 1941.

575 068.—Process for attaching electrodes to piezo-electric elements of the fusible or multi-plate type.

The Brush Development Co. (assignees of C. K. Gravley). Convention date (U.S.A.) 16th December, 1942.

575 223.—Combination of a standard-frequency oscillator with a continuously-variable oscillator in order to allow the "spot measurement" of a drifting frequency.

H. J. Finden and The Plessy Co. Ltd. Application date 8th March, 1944.

575 252.—Arrangement of an auxiliary priming capacitor across the glow-discharge tube of a detonating or like relay.

D. Weighton and Pye Ltd. Application date 25th February, 1943.

575 256.—Impedance bridge which is unresponsive to voltages that are up to 90° out-of-phase with those to be measured.

Honeywell-Brown Ltd. Convention date (U.S.A.) 24th October, 1942.

575 411.—Process for forming selenium "wafers" as used for electric rectifiers and light-sensitive cells.

Standard Telephones and Cables Ltd. (assignees of O. Saslaw). Convention date (U.S.A.) 24th April, 1942.

575 447.—High-frequency testing equipment of the type in which a signal-generator is coupled through an attenuating circuit to the apparatus under test.

W. L. Watton. Application date 17th December, 1943.

575 463.—Magnetometer bridge with accessories for measuring the total intensity as well as the horizontal and vertical components, say of the earth's magnetic field.

H. Hughes and Son Ltd., E. Smith and A. J. Hughes. Application date 18th March, 1944.

575 719.—Construction designed to reduce the weight of a power-supply transformer, particularly for use on aircraft.

Standard Telephones and Cables Ltd. (assignees of F. W. Edmonds). Convention date (U.S.A.) 10th April, 1943.

575 805.—Method of coupling and calibrating a series arrangement of piezo-electric oscillators in order to increase the power that the resulting unit can handle.

B. Tenenbaum. Application date 6th April, 1944.

576 097.—Process and apparatus for bonding together the layers of plywood by high-frequency current.

The General Electric Co. Ltd. and E. McP. Leyton. Application date 5th October, 1942.

576 192.—Optical method of preparing long calibration-strips for use with an oscillator of wide frequency-range.

Standard Telephones and Cables Ltd. and I. V. Feldhusen. Application date 14th April, 1944.

576 473.—Cathode-heating switch having a bi-metallic strip under a "click" control for the sharp operation of a glow discharge tube.

The General Electric Co. Ltd., W. G. Branson and R. W. Strong. Application date 14th May, 1943.